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Two Iconicity Notions in Peirce's Diagrammatology

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Two different concepts of iconicity compete in Peirce's diagrammatical logic. One is articulated in his general reflections on the role of diagrams in thought, in what could be termed his diagrammatology – the other is articulated in his construction of Existential Graphs as an iconic system for logic representation. One is operational and defines iconicity in terms of which information may be derived from a given diagram or diagram system – the other has stronger demands on iconicity, adding to the operational criterion a demand for as high a degree of similarity as possible.

This paper investigates the two iconicity notions and addresses some of the issues they involve.

Operational iconicity

The basic concept of iconicity in Peirce's semiotics and logic is presented in his second tricotomy of sign types, the well-known distinction between icons, indices, and symbols, respectively.[i] This tricotomy deals with the relation between the sign and its dynamic object, and the idea is that this relation may take three different forms. Icons function by means of a similarity between the sign and the object, or, as Peirce may also say, by shared characteristics between the sign and its object. Indices function by means of an actual connection between the sign and its object, either of a causal character (the footprint on the beach) or of a purposive character (deictics, pronomina or proper names in language). Symbols, finally, function by means of a habit, in mind or in nature, of connecting two otherwise unconnected entities to a sign. It should immediately be added, that the sign types of this tricotomy, just as is the case in the later Peirce's other nine tricotomies, do not correspond directly to distinct, natural kinds of signs. They rather pertain to *aspects* of signs, so that pure icons, indices, and symbols, respectively, may be conceived of as borderline cases only, while most typical, and indeed most interesting signs involve all three aspects to different degrees. It is possible, though, in many cases, to point out which of the three aspects is *basic* in a given sign or a given sign type – so as for instance diagrams being basically icons, and only secondarily (but still necessarily) having also indexical and symbolical aspects.

In this basic iconicity definition by similarity or shared characteristics, however, none of the two iconicity concepts to be discussed here, is obvious. They only appear when a further determination of similarity is attempted. The first, operational,

definition appears exactly in the discussion of diagrams, and is developed by Peirce already in the 80s, even if the full articulation of it awaits Peirce's mature philosophy of the years after the turn of the century. To continue in Peirce's detailed taxonomy of signs from that period, icons come in three subtypes, images, diagrams, and metaphors, respectively. Images are to be taken in a special, technical sense not corresponding to our everyday image notion: they are icons whose similarity functions by means of simple qualities only, colour, sound, shape, form, etc. Thus, images are very simple icons, functioning by one or few such qualities only. The recognition of a crescent form as a sign for the moon may serve as an example. The simplicity of images is made clear by their contrast to diagrams. Diagrams are skelettal icons, representing their object analyzed into parts among which "rational relations" hold, be they explicit or implicit. Such relations may be spatial, logical, mathematical, or any other type which may make clear the kind of relation holding between parts. So, as soon as the icon consists of parts whose relations mirror the relations between the corresponding parts of the object, and the sign is used to gain information about those parts and their relations, a diagram is at stake.[ii] In contrast to the technical notion of image, being much more narrow than the everyday use of the word, Peirce's technical notion of diagram is much more wide than the everyday diagram notion: it must include any use of, e.g. a painting, in which the relation between its parts plays a role in the interpretation – and it must include also algrabraic notations which may not, at a first glance, seem diagrammatical. Metaphors, to finish this tricotnomy, are icons functioning through the mediation of a third object, so as for instance an ancestral tree, charting family relationships in a branching diagram structure through the intermediate icon of a tree. The important notion here is the very wide sense of the notion of diagram which stems, in fact, from the operational criterion for iconicity. An icon is a sign "... from which information may be derived.", Peirce says ("Syllabus", ca. 1902, CP 2.309), and this forms the basic idea in the operational criterion: icons as the only sign type able to provide information which is why all more complex sign types must involve or lead to icons in order to convey information. Later in the same paper, Peirce adds that "An Icon, however, is strictly a possibility involving a possibility ..." (CP.2.311), and in this enigmatic formula, the first "possibility" should be read as referring to an icon being a possible sign of everything which resembles it in the respect so highlighted (only an index may make explicit which object or class of objects the sign more precisely refers to, so only the combination of icon and index holds the possibility of actually conveying information in the shape of a proposition). The second "possibility", however, refers to the fact that the similarity characteristics defined by the first possibility involve ,in themselves, possibilities which are not explicit and which may be further developed:

> "For a great distinguishing property of the icon is that by the direct observation of it other truths concerning its object can be discovered than those which suffice to determine its construction". ("That Categorical and Hypothetical Propositions are one in essence, with some connected matters," c. 1895, CP 2.279).

I have earlier argued (Stjernfelt 2000, Stjernfelt (forthcoming)) that this idea constitutes an epistemologically crucial property of the icon: it is nothing but an

operational elaboration on the concept of similarity. The icon is not only the only kind of sign directly presenting some of the qualities of its object; it is also the only sign by the contemplation of which more can be learnt than lies in the directions for the construction of the sign. This definition immediately separates the icon from any psychologism: it does not matter whether sign and object for a first (or second) glance seem or are experienced as similar; the decisive test for iconicity lies in whether it is possible to manipulate or develop the sign so that new information as to its object appears. This definition is non-trivial because it avoids the circularity threat in most definitions of similarity which has so often been noted.[iii] At the same time, it connects the concept of icon intimately to that of deduction. This is because in order to discover these initially unknown pieces of information about the object involved in the icon, some deductive experiment on the icon must be performed. The prototypical icon deduction in Peirce's account is the rule-governed manipulation of a geometrical figure in order to observe a theorem - but the idea is quite general: an icon is characterized by containing implicit information about its object which in order to appear must be made explicit by some more or less complicated deductive manipulation or experiment procedure accompanied by observation. Thus, Peirce's diagrammatical logic rests on the basic idea that all knowledge, including logical knowledge, indispensably involves a moment of observation. Peirce thus writes, as early as 1885:

> "The truth, however, appears to be that all deductive reasoning, even simple syllogism, involves an element of observation; namely, deduction consists in constructing an icon or diagram the relations of whose parts shall present a complete analogy with those of the parts of the object of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts." ("On the Algebra of Logic. A Contribution to the Philosophy of Notation" (1885), CP 3.363)

This operational criterion makes obvious the breadth of the diagram category within icons. As soon as rationally related parts of an icon is distinguished, and the manipulation of such parts is undertaken, we perform a diagram manipulation, developing some of the implicit possibilities involved in the icon.

A very important use of this operational criterion of similarity is now the appreciation of iconicity where it may not be, at a first glance, obvious. Peirce himself makes this use of the operational criterion when arguing that syllogistic logic or algebra are, in fact, instances of diagrammatical iconicity. In what I believe is Peirce's most detailed account for the diagrammatical reasoning process in general, abstracted from particular diagram systems, he thus argues this point (in "PAP" (1906), a parallel version to "Prologomena to an Apology for Pragmaticism" from the same year), Peirce (1976), p. 317-18):

> "Now necessary reasoning makes its conclusion *evident*. What is this "Evidence"? It consists in the fact that the truth of the conclusion is *perceived*, in all its generality, and in the generality of the how and the why of the truth is perceived. What sort of a Sign can communicate this Evidence? No index,

surely, can it be; since it is by brute force that the Index thrusts its Object into the Field of Interpretation, the consciousness, as if disdaining gentle "evidence". No Symbol can do more than apply a "rule of thumb" resting as it does entirely on Habit (including under this term natural disposition); and a Habit is no evidence. I suppose it would be the general opinion of logicians, as it certainly was long mine, that the Syllogism is a Symbol, because of its Generality. But there is an inaccurate analysis and confusion of thought at the bottom of that view; for so understood it would fail to furnish Evidence. It is true that ordinary Icons, - the only class of Signs that remains for necessary inference, - merely suggest the possibility of that which they represent, being percepts *minus* the insistency and percussivity of percepts. In themselves, they are mere Semes, predicating of nothing, not even so much as interrogatively. It is, therefore, a very extraordinary feature of Diagrams that they *show,* - as literally *show* as a Percept shows the Perceptual Judgment to be true, - that a consequence does follow, and more marvellous yet, that it *would* follow under all varieties of circumstances accompanying the premisses."

Here, the operational criterion is used in order to include traditional syllogistic reasoning within the field of diagrams: the structure of syllogism simply *is* a diagram, even when presented in the clothing of ordinary language. The same criterion was early used by Peirce in order to include algebra as icons, even as involving icons "par excellence" in the patterns of manipulation permitted:

"As for algebra, the very idea of the art is that it presents formulæ which can be manipulated, and that by observing the effects of such manipulation we find properties not to be otherwise discerned. In such manipulation, we are guided by previous discoveries which are embodied in general formulæ. These are patterns which we have the right to imitate in our procedure, and are the icons par excellence of algebra." ("On the Algebra of Logic. A Contribution to the Philosophy of Notation" (1885), CP 3.363)

Even if Peirce in this very paper tries to develop a notation of logic which, unlike his later entiative and existential graphs, sticks to traditional algebraic representations, he already here acknowledges that such algebraic representations must necessarily be diagrammatic, as measured on the operational criterion of iconicity. Elsewhere, the extends that criterion to include also aspects of linguistic grammar in the diagram category.

This operational criterion of iconicity thus becomes a very strong tool for a Peircean trying to chart the limits of iconicity. Unfortunately, Peirce never went into a further taxonomical exercise in order to chart the possible subtypes of diagrams – the only reference I found in this direction is a brief comment upon the diagram types of maps, algebra, and graphs, respectively.[iv] In any case, the operational criterion forms a very strong argument in a Peircean diagrammatology – yielding the means of a similarity test which is immune against psychologism and any subjective similarity impressions or confusions.

This broad iconicity and diagram criterion is not, however, without any problems. One terminological issue is that the technical, Peircean notion of diagram is now extended to such a degree that the common-sense notion of diagrams vanishes in the haze and seems to constitute only a small subset of the new, enlarged category. Another, more serious problem, is that Peirce still tends to take such diagrams as *prototypical* diagrams in many discussions, generalizing diagram notions taken from them to the whole category of diagrams. This goes, e.g., for his distinction between corollarial and theorematical reasoning, distinguishing conclusions which may be directly read off the diagram, on the one hand, and more difficult inferences requiring the introduction of new entities in the diagram. This distinction is taken from the prototypical diagram case of Euclidean geometrical diagrams where the new entities introduced are helping lines, etc. As Hintikka has argued, however, this distinction may be valid and indeed highly valuable when extrapolated to the more general category of diagrams. The most serious problem, however, in the generalization of the diagram concept, is connected to the lack of a rational sub-taxonomy of diagrams, namely: by which semiotic means should we now distinguish between, e.g. algebraical representations and topological-geometrical representations of the same content, as for instance the graphical and algebraical-arithmetical representations of the same mathematical functions? If the same amount of information may be operationally derived from such representations, they are, to the exact same degree, diagrammatical representations, and Peirce's diagram category offers no means for us to distinguish the particular properties of these different representations.

Optimal iconicity

This problem seems, indeed, to lie behind Peirce's introduction of a second, moredemanding , notion of iconicity. It is well known that Peirce, in the latter half of the 90's, gave up his early attempts from the 80's at an algebra of logic (two versions of which were developed in 1880 and 1885), now preferring the development of graphical systems known as entiative and existential graphs. Especially the development of the latter was seen by Peirce himself as one of his major achievements, and they have been a central inspiration for diagrammatical or multimodal logic of our day, because they involve "iconical" representations which differ highly from algebraical or "symbolical" representation systems of formal logic, e.g. in the Peano-Russell tradition. I place "iconical" and "symbolical" in quotation marks here to emphasize that the use of such words in this context run directly counter to Peirce's operational iconicity criterion. For according to this criterion, such representation systems are indeed diagrammatical and iconical *to the exact same degree*, provided they yield similar possibilities for extracting new information about their object. If the same theorems may be inferred from such systems, they are, on the operational criterion, both of them operationally iconical. And if we take Peirce's two completed systems of "iconical" logic graphs, the Alpha and Beta systems of existential graphs, they have indeed been proved complete and consistent representations of propositional logic and first order predicate logic, respectively. So, in terms of which theorems may be derived from them, the Alpha and Beta graphs are

just as iconical as propositional logic and first order predicate logic, as developed within mainstream formal logic, and vice versa. Peirce's operational iconicity criterion does, it is true, provide the strong insight that these results of mainstream formal logic are *not,* contrary to widespread belief, "symbolical" in the sense that they do not involve iconical representations. They may, of course, be termed "symbolical" understood in the sense that they employ symbols to a larger degree than Peirce's graphs (which also NB employ symbols), but this term may no longer be taken, implicitly, also to imply that they do not contain iconical representations of their object. This is, indeed, a very strong and to some extent counter-intuitive result of Peirce's operational iconicity criterion. But it immediately raises a further question: *what is then the difference between "iconical" and "symbolical" logic representations when it may no longer be expressed in terms of operational iconicity?*

Even if Peirce does not explicitly (at least where I have searched in his writings) pose the question in these terms, this issue is involved in his introduction of a second, stronger iconicity criterion. This takes place especially in his discussion of the conventions used in his Beta system equivalent to first order predicate logic. While the Alpha system required only a sheet of assertion, letters representing propositions, same location of graphs indicating conjunctions, and cuts representing negations, the Beta system adds to these entities further conventions representing quantifications, variables, and predicates. The whole machinery of these issues isintroduced by means of a very simple convention. Predicates with up to three variables (equivalent to functions with arguments in the Fregean tradition) are introduces by means of the verbal/predicative kernel of the predicate written directly on the graph with the corresponding subject slots indicated by blanks to be filled in by symbols for the subjects involved (nouns, pronouns, or proper names). In ordinary text, such blanks are indicated by underlinings such as in "_____ gives _____ to _____" involving three blanks. In the Existential Graphs, similar lines are interpreted as "lines of identity" so that any further determination of the identity of the subjects of these blanks are to be added to the ends of the lines. The very line of identity thus refers to a variable, and the line may branch in order to tie to different slots in different predicates, indicating that the individual(s) referred to by that line has those predicates. The spots at the end of such lines are, consequently, the second convention added: they refer, as indices, to the binding of the variables bearing the predicates in issue. Thus, the whole logical machinery of quantification, variables, and predicates is represented by these very simple means. If a line of identity abuts on the sheet of assertion (or on any evenly enclosed part of it, that is, by 2, 4, 6, ... cuts), then this immediately indicates the existential quantifier of "Something exists which ..." and the three dots are then filled in by the predicates to which the line of identity connects this implicit quantification. Similarly, any such line of identity ending in an unevenly enclosed cut immediately indicates a negative universal quantifier.[v]

In his development of the Beta system, Peirce lays a great emphasis on the fact that the representation of quantification and bound variables by the means of lines of identity is *more iconical* than the representation of the same issues by means of repeated identification of the same bound variables represented by symbols,[vi] so as for instance when he writes that

"A diagram ought to be as iconic as possible, that is, it should represent relations by visible relations analogous to them." ("Logical Tracts, vol. 2", 1903, CP 4.432)

In quotes such as this, it may remain ambiguous which iconicity concept is exactly at stake, but the fact that Peirce considers alternative, more or less iconic, ways of representation of the same propositions and arguments, shows an alternative iconicity concept being considered. Peirce thus considers alternative representation as substitutes for Identity Lines (here "Ligatures" as a concept for systems of Identity Lines meeting across cuts) under the headline of "Selectives":

"A Ligature crossing a Cut is to be interpreted as unchanged in meaning by erasing the part that crosses to the Cut and attaching to the two Loose Ends so produced two Instances of a Proper Name nowhere else used; such a Proper name (for which a capital letter will serve) being termed a *Selective*." ("Prolegomena to an Apology for Pragmaticism" (1906), CP 4.561)

In cases where the web of Lines of Identity in a Beta graph becomes so entangled that it is difficult to survey, some of these lines may be cut, and the identity of the now severed and scattered bits of Identity Line may be secured by the addition of identical symbolical letters to the outermost end of the remaining Identity Line bits. When reading the graph outside-in, the reader must now take note of the quantification indicated by the location of that outermost Identity Line end, remember the letter representing the Selective and identify the more innerly appearances of the same letter with the first quantification. Peirce explicitly regrets the introduction of these Selectives because they lack the iconicity of identity lying in the continuous line connecting the different predicate which this Identity Line takes:[vii]

"[The] purpose of the System of Existential Graphs, as it is stated in the Prolegomena [533], [is] to afford a method (1) as *simple* as possible (that is to say, with as small a number of arbitrary conventions as possible), for representing propositions (2) as *iconically*, or diagrammatically and (3) as *analytically* as possible. [...] These three essential aims of the system are, every one of them, missed by Selectives." ("The Bedrock beneath Pragmaticism" (2), 1906, CP 4.561 n.1)

The substition for the Identity Line by Selectives is less iconic because it requires the symbolic convention of identifying different line segments by means of attached identical symbols. The Identity Line, on the other hand, is immediately an icon of identity because it makes use of the continuity of the line which so to speak just stretches the identity represented by the spot – and which is, at the same time, a natural iconical representation of a general concept:

"The second aim, to make the representations as iconical as possible, is likewise missed; since Ligatures are far more iconic than Selectives. For the comparison of the above figures shows what a Selective can only serve its purpose through a special habit of interpretation that is otherwise needless in the system, and that

makes the Selective a Symbol and not an Icon; while a Ligature expresses the same thing as a necessary consequence regarding each sizeable dot as an Icon of what we call an "individual object"; and it must be such an Icon if we are to regard an invisible mathematical point as an Icon of the strict individual, absolute determinate in all respects, which imagination cannot realize." (ibid.)

The Peircean Selective, of course, does exactly the same as quantification with bound variables undertake in the traditional system: the first presentation of the variable determines the quantification of it, and later occurrences of that variable in the logical expression remains under the scope of that quantifier. But it remains a second-rate, anti-iconic representation when one and the same bound variable is no longer represented by one entity only (the line of identity) but is, instead, represented by a series of different lines of identity identified only by the addition of symbolical indices, or, as in ordinary formal logic, by the series of x's or y's, identified only by their merely symbolical identity.

The reason why Peirce considers the introduction of Selectives at all is, of course, that in sufficiently complicated Beta graphs involving many variables taking many predicates, the network of Identity Lines may form a thicket hard to get a simple visual grasp of. The reason for introducing Selectives is thus heuristic and psychological, pointing to the specific competences and limitatins of a human observer; we might imagine a mind better equipped than ours which would be able to survey in one glance any complicated web of Identity Lines without having to resort to Selectives.

But the important issue here is Peirce's very motivation for preferring Identity Lines to Selectives in the first place: they are *more iconical*, because they represent in one icon entity what is also, in the object, one entity. This thus forms an additional, stronger iconicity criterion in addition to the operational iconicity criterion. One could object that Peirce was in no position to know the informational equivalence between his Beta system and what was only later named first order predicate logic – but still his argument was implicitly aimed against his own earlier algebraical attempts at logic formalization (a formalization, we should add, which through Schröder yielded a huge impact on Peano's formalization merging with Russell to result in mainstream "symbolic" formal logic). In any case, Peirce realized that the two versions of Beta graphs, with Identity Lines and with Selectives, respectively, was logically equivalent, and the latter even in some cases heuristically superior. And still he preferred the former version in as many cases as possible, thereby indicating a criterion for distinguishing more and less iconical (2) representations among iconical (1) representations being equivalent under the operational criterion. We may indicate these two different concepts of iconicity by iconicity (1), referring to the operational criterion, and iconicity (2), referring to the "more iconical", optimal type of iconicity. Peirce's arguments pro et con Identity Lines and Selectives display two different constraints on logic representations. What counts for the Selectives was heuristic, practical issues tied to the psychology of the reasoner – obviously a constraint deemed less noble by an avowed anti-psychologist like Peirce. What counts for the Identity Lines is rather an *ontological* argument: the idea that using them, Beta graphs more appropriately depict logical relations *like they really are*, thus adding to the pragmatist operational criterion of iconicity an ontologically motivated extra criterion. According

to this criterion, if two icons are equivalent according to iconicity (1), still the representation which is most iconical according to iconicity (2) must be preferred – if heuristic arguments do not count against it, that is.

This implies that the addition of iconicity (2) to Peirce's iconicity doctrine is connected to his *realism*. It is well known that Peirce's realism developed over the years, such as is documented most famously by his own diamond example from the very birthplace of pragmatism, *How To Make Our Ideas Clear* (1878), to which he returns in *Issues of Pragmatism* (1905) in order to correct what he now sees as a youthful failure. In his early doctrine, he claimed that if a diamond was formed within a bed of cotton and remained there until it was consumed by fire, it would be a mere convention to call that diamond hard, because it was never put to any test. In his mature correction, Peirce says that his earlier idea was nominalist and tied to an actualist conception of being. Now, he refers to the "real possibilities" inherent in the very concept of diamond which implies that it is hard because it *would be* tested hard if subjected to the adequate testing – the hardness of the diamond is not only subject to testing but connected to other pieces of knowledge of diamonds' molecular structure, reflection abilities, heat development during burning, etc. While earlier only admitting subjective possibilities – possibilities due to the fact that we possess incomplete knowledge about the fact in issue (in this sense, it is possible that there are living beings on other planets, because we do not know it is not the case) – Peirce now admit that certain such possibilities also have a *real* character, laws of nature being the most clear expressions of such real possibilities (if I held a stone and let go, the stone would fall to the ground). Peirce's admission of such real possibilities in the latter half of the 90's considerably changes and enriches his concept of thirdness as well as his conception of the pragmatic maxim in terms of *would-be*s. Still, this realism was never really incorporated into his logic graphs.

In Max Fisch's famous charting of Peirce's almost life-long development into a still more extreme – or consequent – realism, the last step, only hinted at in some of Peirce's late writings, was the rejection of material implication – the nomal logical interpretation of the implication *p -> q* according to which it is equivalent to *non-p or q*. Of course, the traditional uneasiness with this interpretation is that according to this interpretation, all cases of p being false automatically render *p -> q* true, in contrast to different versions of strong implication, among those implication in everyday language where p being false rather makes the implication irrelevant than true. Most of his lifetime, Peirce was a strong defender of material implication (under the title of "Philonian", as opposed to "Diodoran" implication, the names stemming from Cicero's reference to two competing Hellenistic logicians), but Fisch is right in indicating that the mature Peirce expressed increasing doubts as to the possible nominalism inherent in material implication, admitting as early as 1898 that it does indeed seems strange that an occurrence of non-lightning should really support the implication that "If it is lightening, it will thunder."[viii]:

> "For my part, I am a Philonian; but I do not think that justice has ever been done to the Diodoran side of the question. The Diodoran vaguely feels that there is something wrong about the statement that the proposition "If it is lightening, it will thunder," can be made true merely by its not lightening." ("Types of Reasoning" (1898), Peirce 1976, 169). .

One even stronger locus of such doubt appears eight years later, and interestingly it addresses the interpretation of exactly the issue of Identity Lines in Beta and Gamma graphs:

> "Second, In a certain partly printed but unpublished "Syllabus of Logic," which contains the only formal or full description of Existential Graphs that I have ever undertaken to give, I laid it down, as a rule, that no graph could be partly in one area and partly in another; and this I said simply because I could attach no interpretation to a graph which should cross a cut. As soon, however, as I discovered that the verso of the sheet represents a universe of possibility, I saw clearly that such a graph was not only interpretable, but that it fills the great lacuna in all my previous developments of the logic of relatives. For although I have always recognized that a possibility may be real, that it is sheer insanity to deny the reality of the possibility of my raising my arm, even if, when the time comes, I do not raise it; and although, in all my attempts to classify relations, I have invariably recognized, as one great class of relations, the class of references, as I have called them, where one correlate is an existent, and another is a mere possibility; yet whenever I have undertaken to develop the logic of relations, I have always left these references out of account, notwithstanding their manifest importance, simply because the algebras or other forms of diagrammatization which I employed did not seem to afford me any means of representing them. I need hardly say that the moment I discovered in the verso of the sheet of Existential Graphs a representation of a universe of possibility, I perceived that a reference would be represented by a graph which should cross a cut, thus subduing a vast field of thought to the governance and control of exact logic.
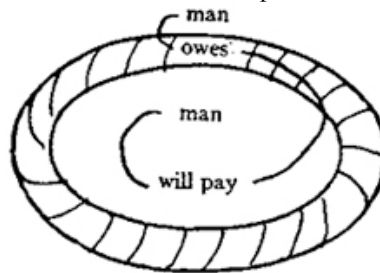> Third, My previous account of Existential Graphs

Fig. 219

was marred by a certain rule which, from the point of view from which I thought the system ought to be regarded, seemed quite out of place and inacceptable, and yet which I found myself unable to dispute. I will just illustrate this matter by an example. Suppose we wish to assert that there is a man every dollar of whose indebtedness will be paid by some man
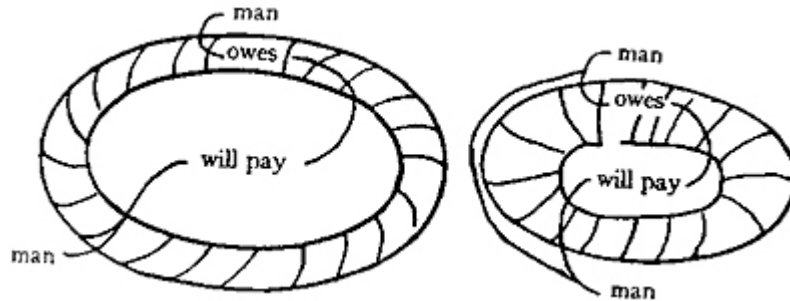
Figs. 220-221

or other, perhaps one dollar being paid by one man and another by another man, or perhaps all paid by the same man. We do not wish to say how that will be. Here will be our graph, Fig. 219. But if we wish to assert that one man will pay the whole, without saying in what relation the payer stands to the debtor, here will be our graph, Fig. 220. Now suppose we wish to add that this man who will pay all those debts is the very same man who owes them. Then we insert two graphs of teridentity and a line of identity as in Fig. 221. The difference between the graph with and without this added line is obvious, and is perfectly represented in all my systems. But here it will be observed that the graph "owes" and the graph "pays" are not only united on the left by a line outside the smallest area that contains them both, but likewise on the right, by a line inside that smallest common area. Now let us consider a case in which this inner connection is lacking. Let us assert that there is a man A and a man B, who may or may not be the same man, and if A becomes bankrupt then B will suicide. Then, if we add that A and B are the same man, by drawing a line outside the smallest common area of the
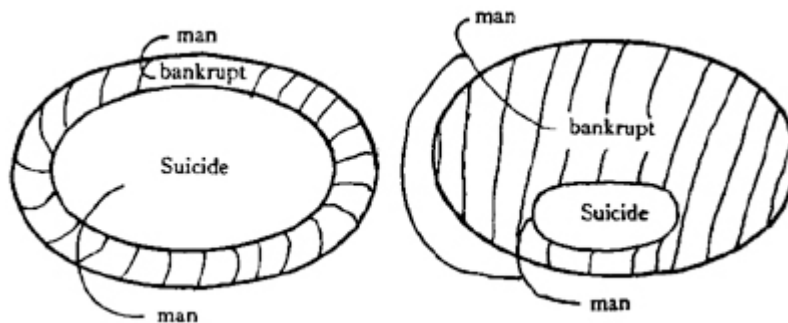
Figs. 222-223

graphs joined, which are here bankrupt and suicide, the strange rule to which I refer is that such outer line, because there is no connecting line within the smallest common area, is null and void, that is, it does not affect the interpretation in the least. . . . The proposition that there is a man who if he goes bankrupt will commit suicide is false only in case, taking any man you please,

he will go bankrupt, and will not suicide. That is, it is falsified only if every man goes bankrupt without suiciding. But this is the same as the state of things under which the other proposition is false; namely, that every man goes broke while no man suicides. This reasoning is irrefragable as long as a mere possibility is treated as an absolute nullity. Some years ago, however, when in consequence of an invitation to deliver a course of lectures in Harvard University upon Pragmatism, I was led to revise that doctrine, in which I had already found difficulties, I soon discovered, upon a critical analysis, that it was absolutely necessary to insist upon and bring to the front, the truth that a mere possibility may be quite real. That admitted, it can no longer be granted that every conditional proposition whose antecedent does not happen to be realized is true, and the whole reasoning just given breaks down.

I often think that we logicians are the most obtuse of men, and the most devoid of common sense. As soon as I saw that this strange rule, so foreign to the general idea of the System of Existential Graphs, could by no means be deduced from the other rules nor from the general idea of the system, but has to be accepted, if at all, as an arbitrary first principle -- I ought to have asked myself, and should have asked myself if I had not been afflicted with the logician's bêtise, What compels the adoption of this rule? The answer to that must have been that the interpretation requires it; and the inference of common sense from that answer would have been that the interpretation was too narrow. Yet I did not think of that until my operose method like that of a hydrographic surveyor sounding out a harbour, suddenly brought me up to the important truth that the verso of the sheet of Existential Graphs represents a universe of possibilities. This, taken in connection with other premisses, led me back to the same conclusion to which my studies of Pragmatism had already brought me, the reality of some possibilities. This is a striking proof of the superiority of the System of Existential Graphs to either of my algebras of logic. For in both of them the incongruity of this strange rule is completely hidden behind the superfluous machinery which is introduced in order to give an appearance of symmetry to logical law, and in order to facilitate the working of these algebras considered as reasoning machines. I cannot let this remark pass without protesting, however, that in the construction of no algebra was the idea of making a calculus which would turn out conclusions by a regular routine other than a very secondary purpose. . . .” (”For the National Academy of Sciences, 1906 April Meeting in Washington”, CP 4.579-81)

In this long quotation, Peirce considerably revises the whole foundation of Beta and Gamma graphs. Cuts no longer represent negation, but merely possibility – they only represent negation if they enclose a further blank cut (meaning everything can be derived from the contents of the first cut, evidently making those contents false). Furthermore, material implication is given up or at least relativized: not all conditional propositions with false antecedents are true. References as relations are included as represented by graphs connecting actuality and possibility, evenly and unevenly enclosed cuts.

Finally, there is the relation between Identity Line conventions and real possibilities which Peirce admitted in his metaphysics from the later 90's onwards (cf.

the diamond discussion). The "strange rule" which Peirce refers to in the quote is presented earlier that very same year and says in its brief form that "... there is some one individual of which one or other of two predicates is true is no more than to say that there either is some individual of which one is true or else there is some individual of which the other is true." ("Prolegomena to an Apology for Pragmatism", 1906, CP 4.569). Now, this rule will imply that the two graphs representing "if A becomes bankrupt, B will suicide", and "if A becomes bankrupt, A will suicide", are identical. Both are falsified if every man goes bankrupt without any man suiciding. However, the two propositions are, evidently, not identical, A and B being potentially different persons in the former proposition, not so in the latter. But the "strange rule" exactly makes of such possibilities mere "nullities". Peirce's hasty and difficult reasoning at this point must refer to the fact that the possibility of A and B being identical is not a mere subjective possibility but a real possibility, given by the possible causal link between bankruptcy and suicidal tendencies, constituting a real tendency in social life.

The fact that it is the very system of Existential Graphs which leads Peirce to these conclusions is taken to count among the chief virtues of that system. While his own algebras hid such facts behind "superfluous machinery" constructed with their (secondary) aim as reasoning machines, the Existential Graphs are not so constructed, but with the aim of displaying to the highest degree of detail and clarity every single logical step taken in reasoning. The efficiency of the algebras is thus contrasted to the logical detail of the graphs – this is an argument referring to the larger degree of iconicity (2) of the graphs, even if they may be equivalent as reasoning machines, that is, with respect to iconicity (1).

This also leads to a further reinterpretation of the iconicity inherent in Identity Lines:

"The System of Existential Graphs recognizes but one mode of combination of ideas, that by which two indefinite propositions define, or rather partially define, each other on the recto and by which two general propositions mutually limit each other upon the verso; or, in a unitary formula, by which two indeterminate propositions mutually determine each other in a measure. I say in a measure, for it is impossible that any sign whether mental or external should be perfectly determinate. If it were possible such sign must remain absolutely unconnected with any other. It would quite obviously be such a sign of its entire universe, as Leibniz and others have described the omniscience of God to be, an intuitive representation amounting to an indecomposable feeling of the whole in all its details, from which those details would not be separable. For no reasoning, and consequently no abstraction, could connect itself with such a sign. This consideration, which is obviously correct, is a strong argument to show that what the system of existential graphs represents to be true of propositions and which must be true of them, since every proposition can be analytically expressed in existential graphs, equally holds good of concepts that are not propositional; and this argument is supported by the evident truth that no sign of a thing or kind of thing -- the ideas of signs to which concepts belong -- can arise except in a proposition; and no logical operation upon a proposition can result in anything but a proposition; so that non-propositional signs can only

exist as constituents of propositions. But it is not true, as ordinarily represented, that a proposition can be built up of non-propositional signs. The truth is that concepts are nothing but indefinite problematic judgments. The concept of man necessarily involves the thought of the possible being of a man; and thus it is precisely the judgment, "There may be a man." Since no perfectly determinate proposition is possible, there is one more reform that needs to be made in the system of existential graphs. Namely, the line of identity must be totally abolished, or rather must be understood quite differently. We must hereafter understand it to be potentially the graph of teridentity by which means there always will virtually be at least one loose end in every graph. In fact, it will not be truly a graph of teridentity but a graph of indefinitely multiple identity.
We here reach a point at which novel considerations about the constitution of knowledge and therefore of the constitution of nature burst in upon the mind with cataclysmal multitude and resistlessness." (op.cit., CP 4.583-84)

All Identity Lines are now to be considered implicitly polyadic – for the realist reason that the entities referred to may have other predicates in common than the ones explicitly mentioned in the graph, thus sharing real possibilities which are not referred to in the explicit graph. Peirce never consistently revised the Graphs according tothe cataclysms of ideas proposed here, but it is obvious that the revisions proposed pertain to the overall idea of iconicity (2) – the attempt at making the graphs match ontological structure to as large a degree as possible.

The pragmatic maxim and the two iconicity notions

The coexistence of two competing iconicity criteria in the mature philosophy of Peirce raises further questions. What about the pragmatic maxim, Peirce's basic idea that the content of all conceptions may be exhausted by considering which practical effects those conceptions would be conceived to have under imagined circumstances? The operational iconicity criterion seems moulded after the pragmatic maxim due to the reductivist action foundation of both: anything which does not have conceived consequences, practically or theoretically, may be discarded. The investigation of possible practical consequences in the former case mirrors the investigation of possible theorems to be inferred in the latter. But this interpretation leaves iconicity (2) in a strange vacuum. If optimal iconicity remains without any practically conceivable consequences, it may be thought to belong to what may be discarded by the maxim as superfluous verbiage. For is there any conceivable practical difference between Identity Lines and Selectives in Existential Graphs? Of course there is the realist conviction that Identity Lines may refer to real generals which may be easier grasped (in some cases, at least) by Identity Lines than by Selectives? And of course there is the practical issue that in complicated cases, Selectives may facilitate an easier use of the graphs than Identity Lines. But at the same time, the amount of theorems, of new information, accessible by the two means are supposed to be exactly the same? Maybe, this difference corresponds to two different readings of the pragmatic maxim, cf. Peirce's own two readings without and with the hardness of the

untested diamond, respectively. The untested diamond hardness and the realist interpretation of the pragmatic maxim seems to correspond to the addition of iconicity (2) as a criterion with its possibilities for distinguishing between more and less iconical representations in addition to the provision of new information, while the earlier, nominalist idea corresponds to the version of the maxim where it charts testable regularities and nothing more. Just like existence is no predicate, it seems like Peircean reality is no predicate neither, and the addition of reality does not add to the amount of information which may be taken out of any given predicate. But Iconicity (2) may add, in some cases, to the heuristics of working with representation systems, just like it presents the same information in a so to speak ontologically more valid form. If that interpretation is correct, then the introduction of iconicity (2) as a criterion constitutes yet another step in Peirce's lifelong movement towards realism, as charted by Max Fisch. In that case, Iconicity (2) is tightly interwoven with the step leading from the Real Possibilities introduced in the latter half of the 90's as the central mode of Thirdness on the one hand, and to Peirce's final and most realist position in search for stronger implications than material implication in the years after 1900, on the other hand.

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NOTES

[i] The tricotomy is the second out of Peirce's three major tricotomies, referring to the sign's relation to itself, to its object, and to its interpretant, respectively. In Peirce's more developed series of ten tricotomies from his later years, it is the fourth.

[ii] It is important to note that Peirce's distinctions pertain to sign *use* rather than to the specific sign vehicles, based on his dictum "A sign is only a sign *in actu* ..." ("Truth and Falsity and Error," *Dictionary of Philosophy and Psychology*, ed. J.M. Baldwin, pp. 718-20, vol. 2 (1901); CP 3.569). Thus, the very same sign token may be used in some contexts as an image – paying no attention to what can be learnt from the relation between its parts – and in other contexts as a diagram. If, for instance, we took the crescent shape, image of the moon, and performed observations on it pertaining to the relation between its parts, if we, say, measured its area in comparison to the implicit full moon area, we would treat exactly the same sign token as a diagram.

[iii] It is an interesting fact in the history of science that such attacks on the notion of similarity have come from otherwise completely opposed camps, namely the analytical tradition (.e.g. Nelson Goodman) on the one hand, and the (post-) structuralists in the continental tradition on the other (e.g. Umberto Eco). See Stjernfelt (2000a) and Stjernfelt (forthcoming).

[iv] In "On Quantity" (ca. 1895, in Peirce 1976, p. 275).

[v] Peirce had already, in his algebras of logic and independently of Frege, invented the "symbolic" quantifier notion. Peirce's version became later, through Schröder and Peano, the standard notation of $\forall$ and $\exists$ (in Peirce's version $\prod$ and Σ, respectively).

[vi] The issue of the iconicity of different aspects and conventions of Existential Graphs is far wider than the alternative between Identity Lines and Selectives which is chosen as the main case in our context because Peirce himself highlights it so thoroughly. The overall iconical motivation in the construction of the graphs is well indicated by Peirce when introducing the details of the graphs:

"I dwell on these details which from our ordinary point of view appear unspeakably trifling, — not to say idiotic, — because they go to show that this syntax is truly **diagrammatic**, that is to say that its parts are really related to one another in forms of relation analogous to those of the assertions they represent, and that consequently in studying this syntax we may be assured that we are studying the real relations of the parts of the assertions and reasonings; which is by no means the case with the syntax of speech." (MS 514, "Existential Graphs" (1909), quoted from John Sowa's commented version of that text).

Shin (2002, 53-58) lists three basic iconical features of Beta graphs, namely Identity Lines, quantifiers and scope. Quantifiers do seem to come naturally because the end of an Identity Line in an unenclosed graph is simply taken to mean "something is ...", but it deserves mention that in Peirce's earlier formalization attempt from the 90's known as Entiative Graphs, in many respects dual to Existential Graphs, the very same sign is taken to stand for the universal quantifier. Maybe it could be argued that a point in a plane does indeed more naturally mean "something" than "all". Scope seems to come natural in the endoporeutic, outside-in, reading of the graphs (which Shin is otherwise out to dismantle), because the outermost occurrence of part of an Identity Line defines the scope of the corresponding quantifier, and more innerly located quantifiers are taken to lie within the scope of the more outerly ones.

In addition to these iconicities, a basic iconicity in Existential Graphs is one of its very motivating ideas in Peirce, namely the representation of material implication by means of a "scroll", that is, two nested cuts where the premiss is placed within the outer cut but outside the inner cut, while the conclusion is placed in the inner cut. This geometrical inclusion of the conclusion within the premiss furnishes a simple iconic representation of the idea that the

conclusion lies in, is inherent in, or is im-plicated by the premiss. Peirce proudly refers to this in CP 4,553 n1 (from "The Bedrock beneath Pragmaticism", 1906) while at the same time complaining about the lack of iconic representation of *modality* in the Graphs, a lack he attempts to remedy not much later, cf. below.

Another issue discussed by Shin – but not in relation to iconicity – is Peirce's distinction between logic systems as result-oriented calculi and logic systems as representations of logical thought process (a distinction she strangely thinks loses its relevance in graphical systems). Here, the former aims at quick and easy results, and a plurality of logical connectors and rules may be used to further that aim as expediently as possible. In the dissection of logical inference steps, on the other hand, as few connectors and rules as possible should be chosen, in order to be able to compare the single steps taken – a guideline explicitly followed in Peirce's graphs. In this connection, Peirce remarks that it is "... a defect of a system intended for logical study that it has two ways of expressing the same fact, or any superfluity of symbols, although it would not be a serious defect for a calculus to have two ways of expressing a fact." ("Symbolic Logic", in *Baldwin's Dictionary*, 1901/1911, CP 4.373). This requirement – which Existential Graphs do not perfectly satisfy – is obviously iconical, demanding the extinction of arbitrary, that is, non-iconical, choices between parallel representations.

Finally, Pietarinen's (forthcoming, 128-31) argument against Shin runs along these lines: her rewriting of the inference rules of Peirce's graphs gives many more rules and connectors than does Peirce's own version, and so is less analytical and iconical than his (even if maybe facilitating easier readability on some points). In his defense of the endoporeutic, outside-in, interpretation of the graphs against Shin's attacks, Pietarinen highlights a further and very basic iconical feature in them: the dialogic structure, rhythmically changing between a Graphist and a Grapheus, responsible for existentially and universally quantified propositions, respectively, and thus responsible for taking turns in a dialogue where each of them manipulates the graph according to Peirce's rules. Pietarinen of course makes this point in order to facilitate his interesting, Hintikkan interpretation of the graphs in terms of game-theoretical semantics, where the two interlocutors hold opposed atrategic aims in the conversation: the proof or disproof of the initial proposition, respectively.In our context, we may emphasize the basic iconicity inherent in this conversational structure of the graphs, motivated in the supposedly dialogical structure of thought, be it between persons or between positions in one person's thought and mind.

[vii] Given the equivalence between Identity Line and Selective representations, we might use this idea in reconsidering ordinary Peano-Russell-style formal logic – here, we might see the different instances of the same bound variable in a symbolic expression as invisibly connected by an erased Identity Line running in an additional line parallel to the line of the normal expression.

[viii] Two years earlier, not long before the introduction of Real Possibilities in January 1897, the doubt is awakening: "It may, however, be suspected that the Diodoran view has suffered from incompetent advocacy, and that if it were modified somewhat, it might prove the preferable one." ("The Regenerated Logic", 1896, CP 3.442-3). But as early as the second "On the Algebra of Logic" (1885, 3.374), Peirce states that "If, on the other hand, A [the premiss] is in no case true, throughout the range of possibility, it is a matter of indifference whether the hypothetical be understood to be true or not, since it is useless. But it will be more simple to class it among true propositions, because the cases in which the antecedent is false do not, in any other case, falsify a hypothetical." Here, Peirce observes the problem, but accepts material implication out of simplicity (and not iconicity) reasons.