Diagrammatic Problem solving

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Diagrammatic Problem solving

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In this paper, we shall look at types of problems and their interplay with diagrammatic representation. The general notion of *diagram* goes back to Peirce (e.g. "PAP", in Peirce 1976, 316ff), see (Stjernfelt 2007 ch. 4) for an in depth introduction. Here, we will focus upon prototypical diagrams - a two dimensional, stylized topological (geometric) representation of some subject matter. In some cases this representation is analogical as for instance in a map of a country, but in other cases it is a geometric representation of quantitative relations as for instance in a pie chart diagram. The difference is whether we map an already existing geometric relation onto the diagram or whether it is quantitative or other relations which are geometrized. If we add time as a dimension to the traditional spatial dimensions, we can also interpret temporal relations as geometric so for instance a flowchart diagram of information currents in a company will qualify as a diagram of geometric relations. However, there are other relations than spatial/temporal and quantitative; for instance interpersonal relations: a crime investigator might set up a diagram of possible interpersonal relations between the different suspects and associates in order to get an overview of the problem. In short: a prototypical diagram is a two-dimensional geometric representation of something we may qualify as “relations” which might then be spatial/temporal relations, quantitative relations, interpersonal or other relations.

When we have a diagram then it is possible to explore it. This exploration can take two forms 1) the exploration does not add anything to the diagram but consists in an examination of possible true statements that may be deduced directly from the diagram. This is the case of most information in both the pie chart diagram and the map of the country. From the map we can read the distance between A and B (given the map scale), find the shortest route between A and B etc. without adding anything further to the map.[1] 2) In the second type of explorations there is a manipulation of the diagram. In the example with the crime investigation the detective might add some connections in the diagram just to check if these connections are true what can then be deduced. In Peirce’s theory (e.g. "Minute Logic", 1902, CP 4.233) and (Stjernfelt 2014, ch. 10) these two types of diagrams are called *theorematic* (when there is manipulation or the addition of further elements) and *corollarial* when the information can be read off the diagram directly. This terminology is well known from mathematics where the theorems that the mathematician proves are the hard stuff requiring information that is imported from outside the problem space, often introduced as lemmas, and the corollary is the true statement that can be read off directly from the theorems without further manipulation. It is clear from Peirce’s definition and examples of theorematic reasoning that by ‘manipulation’ he does not just refer to external manipulations of the diagram but also to any mental manipulation so that if the problem solver confronted with the diagram imagines some kind of operation on the diagram it qualifies as theorematic

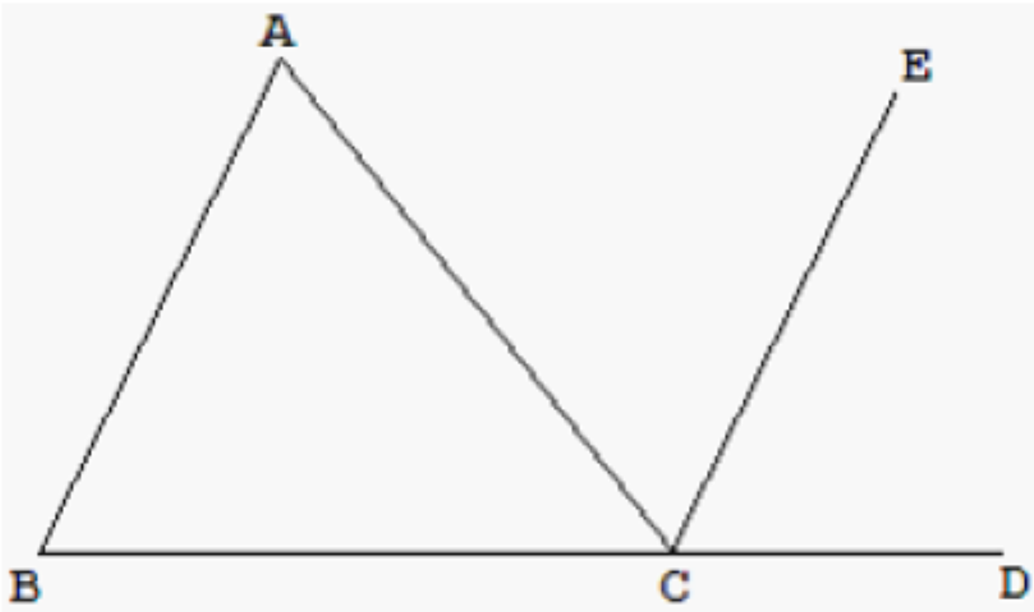
[1] Even in this case, however, it could be argued that something is added - namely the points departure and destination and the line routes explored between them. In this sense, the distinction we are making is rather one of a continuum between less and more manipulation/addition.

reasoning; for instance, in the mundane case of a map one can image things like “if I go this way I get to B” etc. For this reason, theorematic reasoning constitutes a heterogeneous set that can be explored further for internal differences and subtypes; an attempt of this can be found in (Stjernfelt 2014, ch. 10.3).

Solving a problem is, to a large degree, a question of focusing on the right information, and a diagram is a representation of information relevant in the situation. In the following we will briefly present a typology of types of problems and the corresponding diagrams. The diagram is per definition a geometric/topological structure so if the source is not itself geometric the diagram is the result of a mapping from another source domain to a geometric domain and this mapping is performed by humans, so the diagram is not just the visual representation but is in fact indicative of how ways the human mind may work. In other words, thinking is already diagrammatic. This connects to Peirce’s radical claim that solving any mathematical problem necessarily involves diagrammatic manipulation; this is because it is through diagrammatic representation and diagrammatic manipulation that the mind accesses the information necessary to solve any given problem.

Information internal or external to the problem space

The major distinction concerning problem solving is whether all information is present in the diagram or whether one has to add information from outside the problem space. The first is the prototypical case and includes all types of dynamic operations on a given diagrammatic representation. They may be mental or they may involve addition of elements as long as they belong to the problem space. A case of a rule-governed manipulation we find in chess. The position on the board can be considered as a diagram which you can only manipulate following certain rules; the chess player reasons by imagining moving a piece and estimating the possible opponent moves and in this way she might find the optimal solution of the position.



The classical example mentioned by Peirce is Euclid's proof that the sum of the angles in a triangle is 180° a version of which is given above. Given the triangle ABC, you extend the line BC to D, and from C you draw a new line CE parallel with AB. The angle between this new line CE and AC is identical to the angle A and the angle between the new line and the extended line CD is identical to the angle B. So, the three angles meeting in C are the same as the three angles of the triangle, and as BD is a straight line, the three add up to 180°. So, the sum of the angles is 180°. Although two new elements are added, they do not come from outside the problem space, on the contrary they, as well as the triangle, are part of the two dimensional space and they are drawn according to the general axioms of Euclidean geometry - which are analogous to the chess player who imagines moves in accordance with the general rules of chess. In both cases, however, what is additionally required for the problem solver is some strategic information. It is not sufficient to know Euclid's axioms and chess rules - in both cases, some sort of strategy must be pursued: the relevant auxiliary lines to add must be chosen over an infinity of others, and strategically clever moves sequences must be chosen among the finite set of possible moves.

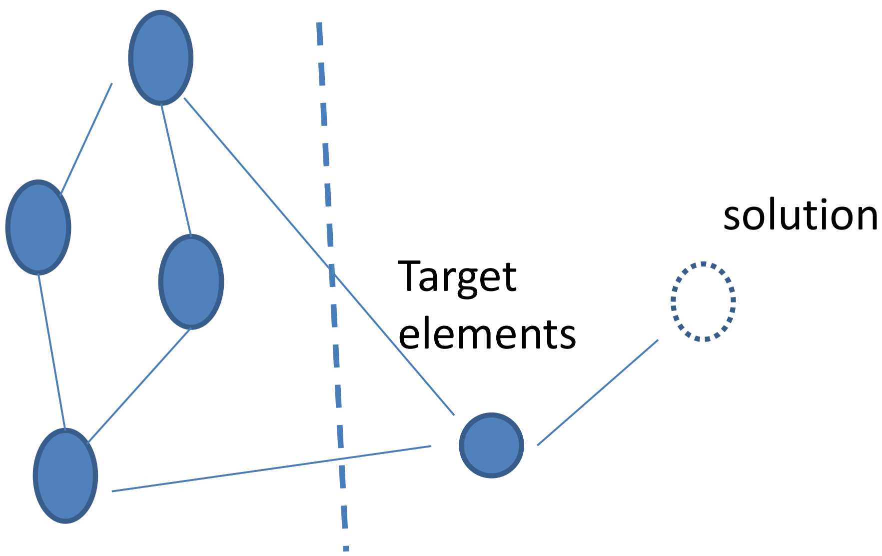
A subclass of problems in the general category of information extraction from the diagram space is what we could call a perspective shift. This might include all cases where there is a reorganization of the elements in a diagram based on abductive reasoning, i.e. a manipulation that is not rule-governed. For instance, the crime investigator might reorganize the relations shown in his diagram of the possible connections between the criminal elements, this might involve a perspective shift where instead of A, B now is considered the main culprit. An example from science could be Copernicus' reorganization of the Ptolemaic model of planetary movements. Instead of taking the sun as the center of the movements Copernicus took an Earth-centered viewpoint, not based on any rules but because he sensed that this could explain the empirical observations better. In (Stjernfelt 2014, 278f) another Peircean example from mathematics is discussed - a proof of Desargues' theorem - where the reorganization consists of embedding the elements in a three-dimensional space instead of two-dimensional.

The type of problems described above contrasts the cases where the information needed is not present in the problem space. For instance, the diagram of the criminal investigator typical contains question marks. Empty slots in the diagram that the investigator tries to fill out by looking for clues he does not yet know about. The typical examples of this group of problems are mathematical problems, though. Mathematical problems will in many cases need helping theorems so-called lemmas. A lemma is itself a theorem of a simpler kind and often not directly connected to the problem in question;

for instance, to know if one can find the solutions to an equation of n^{th} degree one has to know something about the possibilities of permuting a row of n letters, which obviously belongs to another domain than solving an equation.

Insight problems

In the gestalt approach to problem solving the focus is on another distinction, namely whether the solution requires insight or not (Köhler 1925, Duncker 1945, Ohlsson 1992) - which may be seen as a correlate to the theorematic/corollarial distinction. Insight can be defined by whether a specific target element has to be accessed in order to solve the problem or not. Consider for instance the following problem*: Describe how to put 27 animals in 4 pens in such a way that there is an odd number of animals in each pen*. This problem can only be solved by putting all the animals in one pen and place the others in concentric pens around it. The target element is in this case *the diagram of concentric circles*, without accessing that diagram the problem cannot be solved. We can schematically represent the idea of a target element in the following way:



To the left we have the different positions that the problem solver will move between without getting the solution, but she might accidentally cross the boundary and hit the target element on the right, in which case the solution becomes trivial.

The insight problems are opposed, of course, to problems that do not require insight. The border between the two classes is slippery, though; take for instance a chess problem. One can make a program that can solve any problem instantly and this is not done by insight but by systematically going through all possibilities until it hits upon the solution, but for humans the cognitive process that leads to the solution will proceed along strategic schemata and have the characteristics of insight which possibly is measurable in the problem-solving indivdual as an increase of entropy in behavior just before discovering the target move. For instance, in (Metcalfe and Weibe 1987) people's feeling of warmth are recorded while they solve insight and non-insight problems. There was a progressive increase in warmth during non-insight problems. But with insight problems the warmth ratings remained at the same low level until suddenly increasing dramatically shortly before the solution was reached. Similarly, in (Stephen et.al 2009) it is shown that when people solve a gear problem (see below) there is an increase in entropy just before the see the optimal strategy of counting the gears. Here the entropy is measured through eye tracking, i.e. the gaze pattern is stable for a long time, but just before reaching the solution it becomes instable. So it seems that in terms of behavioral dynamics the insight problems are determined by a catastrophic point, namely reaching the target element whereas solving non insight problems is determined by a continuous process reflecting that these problems are routine tasks and tasks where information is used incrementally, as e.g. in the Tower of Hanoi problem (Ohlsson 1992).

We can now classify problems and their relation to diagrams according to whether it is an insight problem or not. If it is not an insight problem, then the diagram is a representation of some subject matter and from this representation it is possible to extract information by making mental operations without fundamentally changing the diagram, as mentioned above. For instance, a prototypical diagram is a flow chart diagram, which is a representation of the temporal dynamics of some domain: water flow in a heating system, information flow in an institution, etc. Given such a diagram, one can read off possible routes in the diagram; similarly in the pie chart diagram one can get direct information about relative size of expenditures in a single domain. However, in these examples, as is often the case, the diagram is a representation of a source domain so one can make experiments on the diagram in order to find alternative organizations of the source: if the heating system doesn't work properly one can look at the diagrammatic representation to solve the problem. Whether the manipulation is relying on insight or not is determined by the underlying process as mentioned above, so if we manipulate a map incrementally to get to the solution it is not insight, but if we have to use some hitherto unknown property of the diagram it may be insight.

The insight problems we propose to divide into three types. Firstly, we have the insight that relies in information that is present in the problem space and which, although not easy to find, do not violate entrenched schematic representation of the field. Solving the problem of finding the sum of the angles in a triangle is an example of this. Remember: insight problems hinges on a target element that gives access to the solution. In the case of the sum of the angles the target element is the line parallel with AB, cf. the figure above. When this line is added to the diagram the problem becomes corollarial because this line provides direct access to the solution. Although not easy to find, the parallel line does not violate any schematic knowledge about geometry, cf. above.

Secondly, we have the problems where the solution requires new elements that are not directly accessible. This includes the cases mentioned above: the criminal investigation where one clue might give the insight to the solution and of course the mathematical problems where some external element might be the clue.

Thirdly, we have an interesting class of problems where the target element might or might not be part of the problem space but where it will in any case constitute a violation of entrenched knowledge or entrenched schematic representations of the world. Entrenched knowledge gives rise to *fixation* according to the gestalt theorists (Duncker 1945). The mind was fixed in a specific representation, which blocked access to the solution. The problem of the pens mentioned above is an example of fixedness of representation, since we tend to think of pens with animals as disjoint. Manipulating the pens in order to solve the problem is a diagrammatic manipulation. In this work one can only solve the problem if one stumbles upon the target element, which in this case is the concentric arrangement of the pens. In (Stjernfelt 2007) we find an interesting example of the same type of problem namely the German geographer and explorer Alfred Wegener's discovery of the plate-tectonics of current geology. Wegener was doing simple diagrammatic manipulation using the information you have on a map of the world and noticing that the West coast of Africa fit with the East coast of South America. This is more than just using the information present in the diagram because it requires the breaking up of the entrenched assumption that the continents were fixed. Geometrically, of course, the manipulation is simple, but geologically, the manipulation broke with entrenched assumptions about the structure of the Earth crust. One can also mention the Copernican revolution as an instance of this, breaking with the entrenched and religiously motivated assumption that the Earth is the center of the universe - or the discovery of non-Euclidean geometry breaking with the assumption that if we have a point outside a line we can draw exactly one other line through the point that is parallel with the given one. Or the definition of imaginary numbers: When Cardano tried to find a solution of an equation of third degree he introduced the square root of a negative number in the formulas for the solution. In order to solve a problem concerning real numbers he had to invent a wholly new type of number outside the domain of real numbers. So this is a case where the diagrammatic manipulation of the equation reaches an obstacle which give rise to a redefinition of our very concept of numbers[2]. This is in many cases a central purpose of diagrammatic reasoning, namely to find the problematic spots in the reasoning process.

To summarize: we have problems that imply simple diagrammatic manipulations and problems that can only be solved by accessing one (or more) target elements, for instance in finding the sum of the triangle there might be more elements that could provide the solution. In the last case the target element might, in some cases, constitute a break with our entrenched knowledge about the world and finally the target element might be part of the problem space as is the case with the four pens and the sum of the triangle or it has to be imported from outside the problem space as is often the case for mathematical problems.

Diagrammatic re-description and diagrammatic re-encoding

[2] An equation is a diagram because the spatial arrangement determines what manipulation can be done.

One of the characteristics of human cognition is the ability to represent a subject matter in an external format; for instance represent the landscape in a map or the country's finance in a pie chart diagram. This is a re-description of a subject matter in a representational format, i.e. a representational re-description. We find this notion in (Karmiloff-Smith, 1992) where the ability to redescribe a subject matter in in another symbolic representation is seen as a basic aspect of the cognitive development of children. As the examples suggest, a diagram is a representational re-description, so the basis for this cognitive ability and thereby the ability to make abstractions is the diagrammatic thinking. The diagrammatic re-description is the key to improve already existing representations. Take as a simple example the decimal number 5,5. In this number there are two fives but they don't mean the same thing. The meaning is determined by the spatial position and for that reason it is an example of a diagram that would replace a more cumbersome representation using fractions. We find another prime example of diagrammatic re-description in the so-called algebraic geometry. The basic idea is that curves can be represented in a symbolic format by equations. For example $x^2 + y^2 = 5$ represents a circle of radius 5. One diagram, a drawn circle, is represented by another diagram, an equation. It is possible to make (algebraic) manipulations with the last diagram and thereby prove any number of properties of curves easier than by the purely geometric methods of the classical Greeks.

The notion of diagrammatic redescription is closely related to the gestalt notion of *re-encoding* (Ohlsson, 1992). In the gestalt tradition re-encoding means that some aspect of the problem representation is reinterpreted. In a diagrammatic representation this means that elements in the diagram are given a new interpretation; for instance, in Alfred Wegener's discovery the continents are re-encoded as being slowly floating instead of being stationary. As the example suggests, re-encoding is mostly a case of insight. The situation is coded according to some entrenched assumptions which block access to the solution. In other words, access to what we have called the target element sometimes requires a re-encoding. In terms of diagrammatic representation this is of course relevant for the problems whose solution relies on perspective shift and the examples mentioned above would be typical for a recoding procedure: recode the problem as embedded in 3D and recode the sun as the center for the planetary moves.

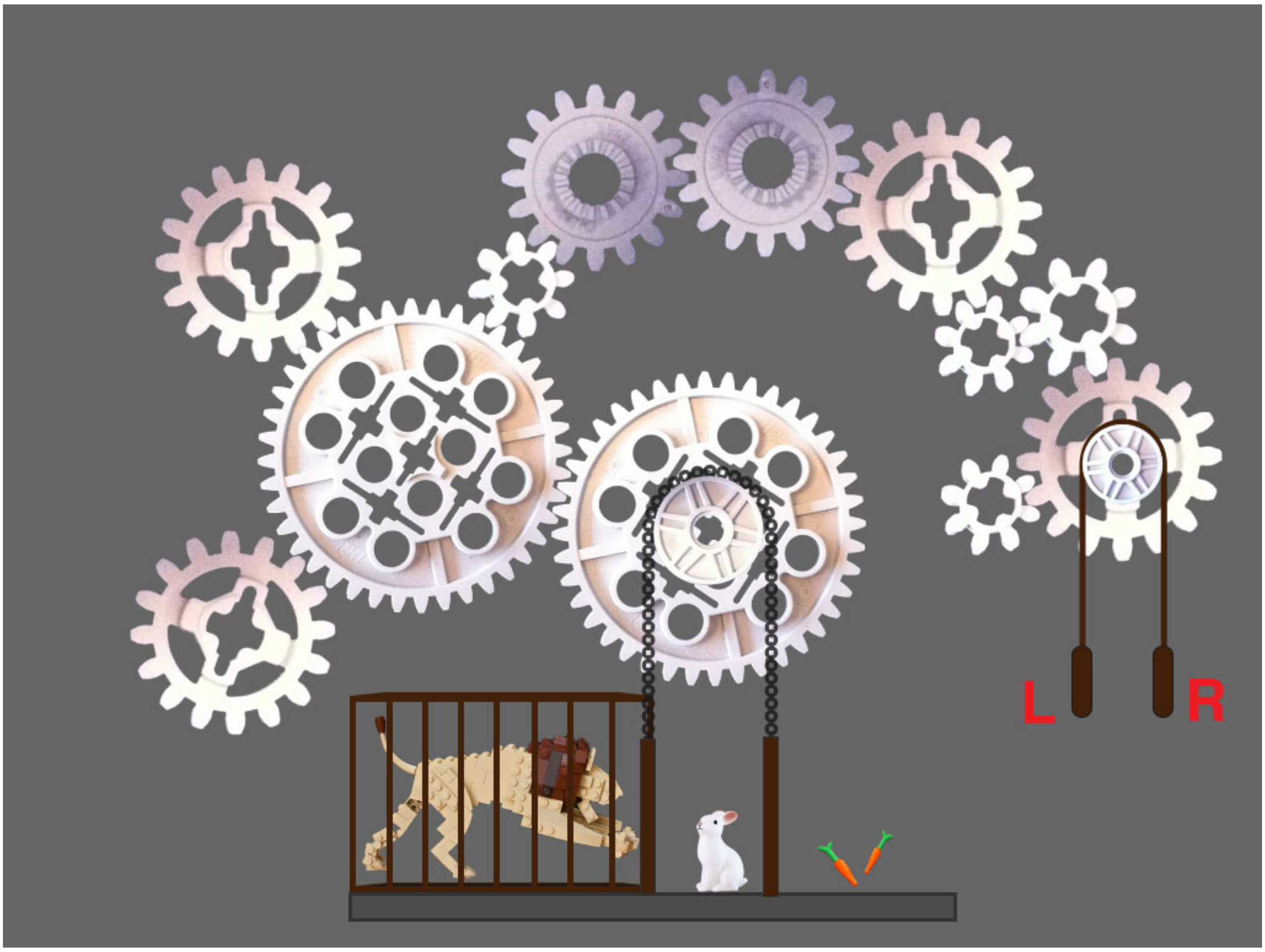
A special case: the Cogwheel experiment

In an experimental setup that is described in more detail below, participants organized in pairs look at a board with a string of connected cog wheels; they then have to determine what way to turn the first cogwheel in order for the last one to turn left or right. This example shows a dual aspect of diagrammatic thinking: on the one hand the problem is presented in the form of a diagram, so the board with the represented cog wheels is no different from a map of a country, a pie chart diagram etc. It is then possible to perform bodily and/or mental operations, i.e. experiments on the diagram. However, in this case the representation the participants make in solving the problem are themselves diagrams; for instance, one pair might make gestures of circles, another might follow the contour of the cogwheels in a continuous curve, etc. We consider all these gestures as diagrammatic representation of a procedure that will lead to a possible solution. The transformations of the gestures during the trials correspond to the manipulations of a diagram whereby the diagram might change considerably. For instance, a participant might start by making circles, alternating between circles turning clockwise and circles turning anti clockwise from wheel to wheel, this may degenerate into a simple wriggling and then finally just a pointing. This corresponds to a process where superfluous information is discarded, and the implied diagram changes appropriately. For instance the process full circle -> wriggling -> pointing corresponds to discarding first the information about the size and turning of the wheels, leading to a simple wriggling indicating direction, then discarding the information about the direction of turning leading to a simple pointing, often accompanied with speak indicating symbolized directions . This corresponds to a process of abstraction, which is a way to make a representation of the subject matter that discards unnecessary information.

Is this case of diagrammatic problem solving an instance of insight or not? The answer is that it may be both. If the solver sticks to the mechanics of the system of wheels then it is easy to solve the problem in a pedestrian way by keeping track of how each individual wheel turns. This is mechanical and does not require any insight; it's like finding one road in a map. The insight requires a perspective shift as in the cases mentioned above, namely from a system that consists of n wheels to a system that only contains two wheels. Every second wheel is categorized as the same so every time we count an odd number of wheels it is the same as the starting wheel and every time we count an even number it is the same as the second wheel. The final insight is then to see that also the number of wheels is unnecessary information. The only necessary information is whether there is an odd or even number of wheels (so the counting may also proceed as "odd-even, odd-even, ..." or "A-B, A-B, ..."). The purpose of the experiment is to see if the participants get to this insight, and if so, how. If they do, to what extent can this be predicted from the gestures (diagrams) they produce from the very beginning.

Cog wheel lessons

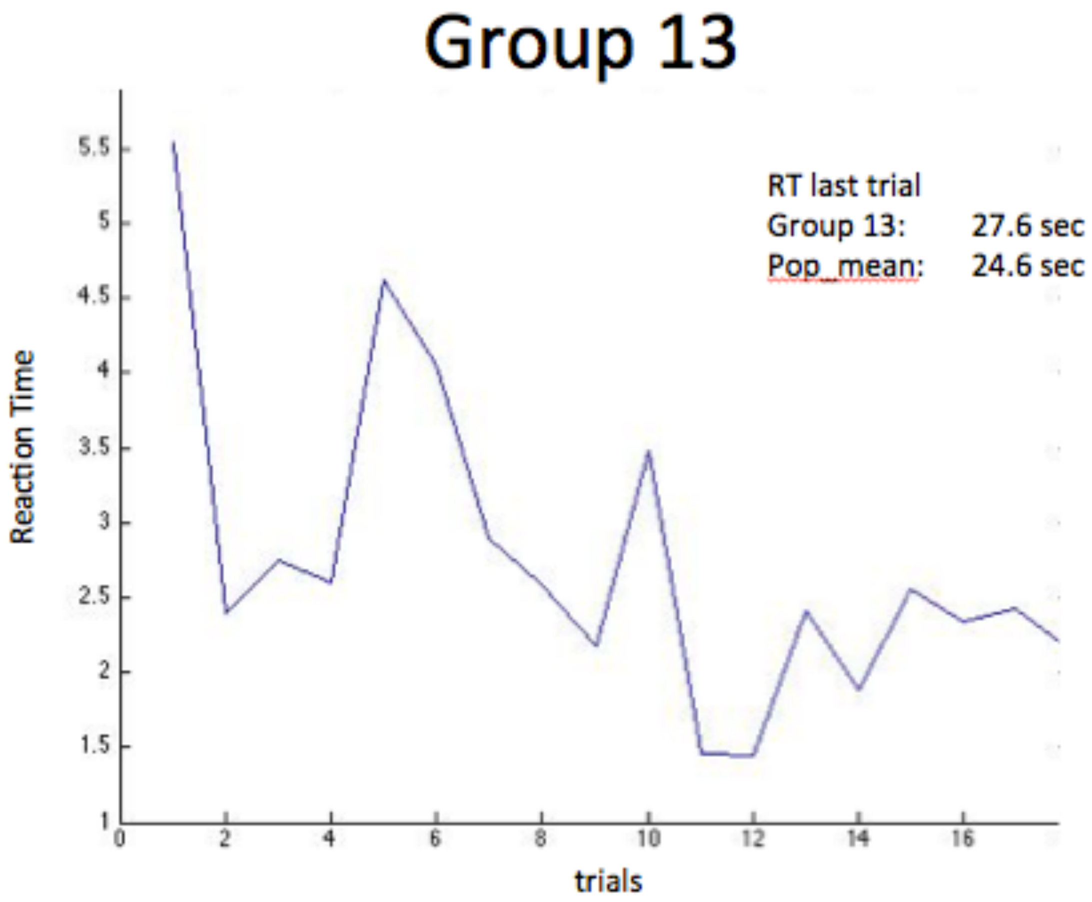
In the following, we shall present some results from the cog wheel experiment realized at the Center for Semiotics in Aarhus in 2012-14. The overall idea is to present a cog wheel problem as described above to two persons in order to investigate their collaboration activity during their solution. The problem was presented to each pair of participants on a 60'' screen on the wall with an empty space before it so that the pair is able to go close to the screen, gesturing and touching it if they so wish. The activities of each pair was then videofilmed and recorded, just like their gestural activity was recorded by measurement devices on their wrists. A simple example of the task is the following:



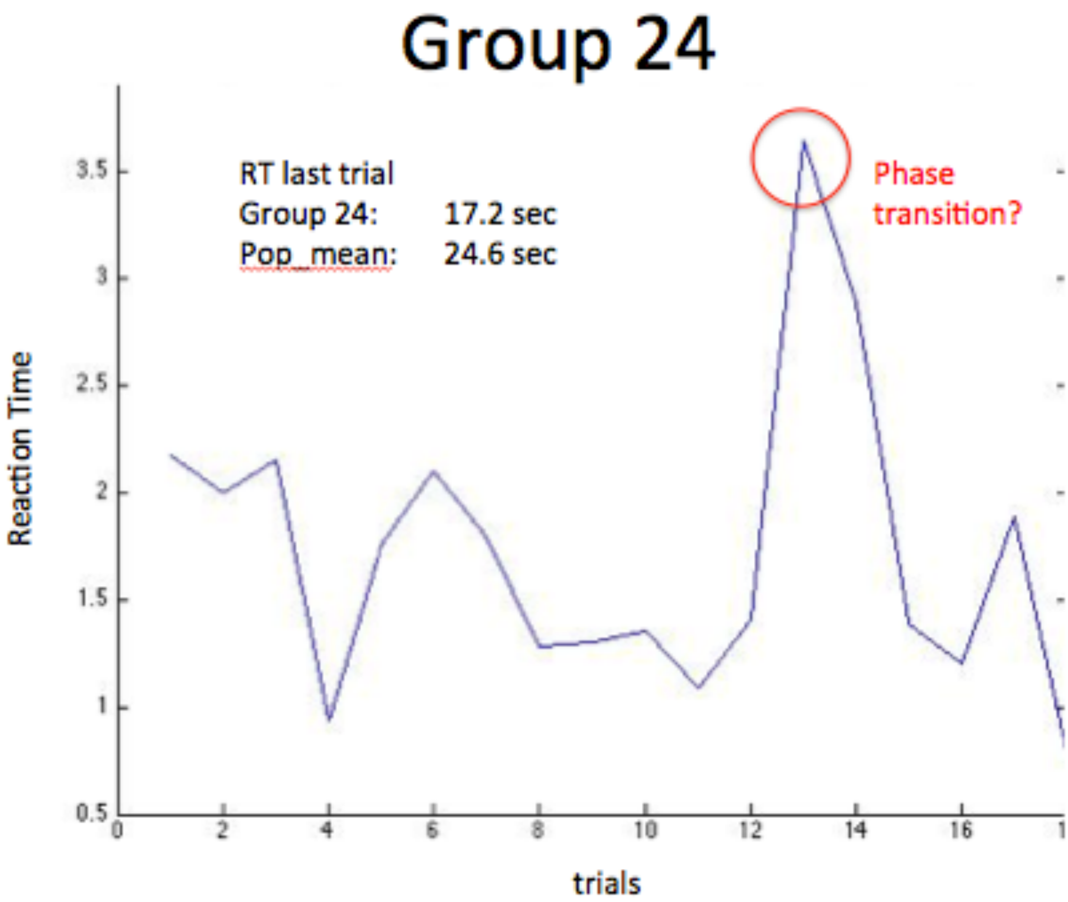
The task is to determine whether to pull the left or right lever in order to give the rabbit access to the carrots rather than to give the lion access to the rabbit. The cog wheel diagram requires an interpretation in which the single wheels are able to move around their center and thus pass energy and resulting movement to connected wheels. This interpretation of the diagram, in itself static, as a dynamical physical system is so obvious that participants immediately adopt it, requiring no explicit instruction in diagram rules in order to begin solving the problem.

In the experimental setup, 25 pairs of participants solved a series of cog wheel tasks. Each pair was subjected to 18 trials with different gear systems, advancing from around 5-6 connected wheels to 13 gears. In a final trial, they were presented with a problem containing 28 gears, in which their solution time was measured. The initial idea behind the experiment was that two different solution strategies are at hand (Stephen et al. 2009) - one continuous and one discrete. The continuous strategy follows the imagined wheel movements from one wheel to the next - while the discrete strategy indicates, in stepwise sequence, the movement of each wheel as the opposite direction of the former. Earlier experiments (ibid.) seem to show that the the former strategy is the most immediate while access to the latter, more efficient strategy takes the shape of a phase transition in the conceptualization of the problem. By presenting the problem to collaborative pairs of participants rather than to single participants, the idea was to investigate whether the phase transition in solution behaviour was primed or accompanied by significant changes in communicative behaviour, gestural or linguistic. Findings proved more complicated, however, indicating that there are indeed several possible phase transitions in solution strategies, combining verbal and gestural behaviour in characteristic ways.

The simplest and to some participants most immediate strategy traces the movement of the initial wheel with a circling finger and then goes on, continuously, to successive wheels. This can be called the "continuous motor strategy", and in the respondents, this strategy is never accompanied by verbal differentiation of direction (like "this way"/"that way", "left"/"right", "up"/"down" etc.)



Group 13 is an example of this strategy. Its final trial solution time is a bit above average. Another strategy could be called the "wiggling hand" strategy. Instead of continuously tracing movement from one wheel to the next, the movement of wheels is indicated by alternate hand movements, typically with the hand a bit removed from the screen. The ‘wiggling hand’ differentiates directions and often leads to effective (abstract) strategies, such as the accompanying of wiggling with verbal differentiation of direction or pointing accompanied by counting.



Group 24 forms an example of the wiggling strategy. A peak in reaction tie around trials 13-14 might indicate problems in solution strategy which are subsequently solved, forming a phase transition to a better strategy - with a resulting last trial significantly below average.

Solution strategies

Looking at the whole population, the types of behaviour are found to have considerably more possibilities than the simple continuous/discrete phase transition orgininally assumed.

Focusing upon gesture, we find no less than 5 qualitatively different hand/arm movement patterns:

> *1. Drawing a full circle for each wheel, typically with just one finger on the screen*
>
> *2. Drawing the half arc of a circle before continuing to the next wheel, typically with one finger.*
>
> *3. Wriggling the open hand left or right over each wheel*
>
> *4. Drawing one continuous curve following the outlines of the gears*
>
> *5. Pointing sequentially to wheel after wheel, typically with one finger*

As to verbal behaviours (apart from coordination talk, meta-talk and jokes etc. among participants), we observe 4 qualitatively different types:

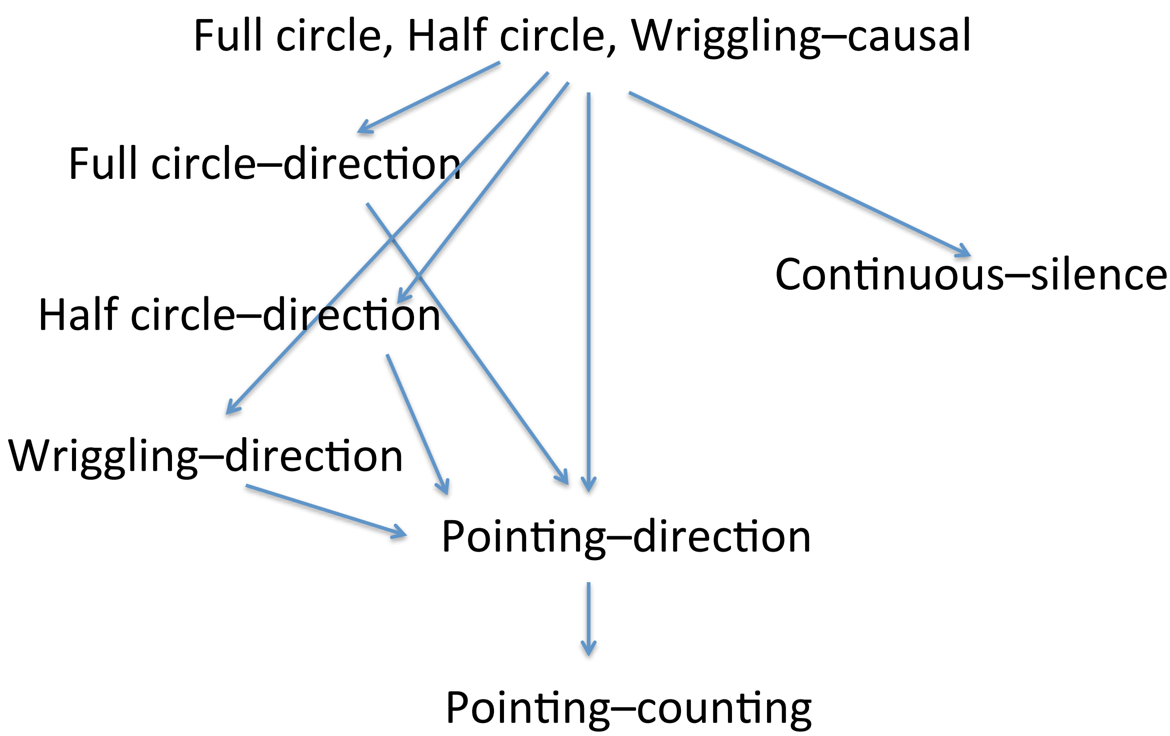
> *1. Causal reasoning: "if we pull this one, then this one goes up", abbreviated to "if this way then this way" or just "this way, this way"*
>
> *2. Describing the alternativng directions of movement of the wheels: "left, right" or "clockwise, anti-clockwise"*
>
> *3. Silence (while gesturing)*
>
> *4. Counting: 1, 2, 3 etc.*

These different gestural and verbal behaviours are found to combine in the following sets of stable patterns:

gesture	**language**
• Chaotic use of gestures	• Causal
• Circle/half circle	• Causal
• Circle/half circle	• Direction
• Wriggle	• Causal
• Wriggle	• Direction
• Continuous curve	• Silence
• Pointing	• Direction
• Pointing	• Counting

These behaviours combine sequentially in a series of typical developments which may be indicated in the following flow chart diagram over the landscape of different phase transtions between stable solution strategies (each arrow indicating a shift in strategy):

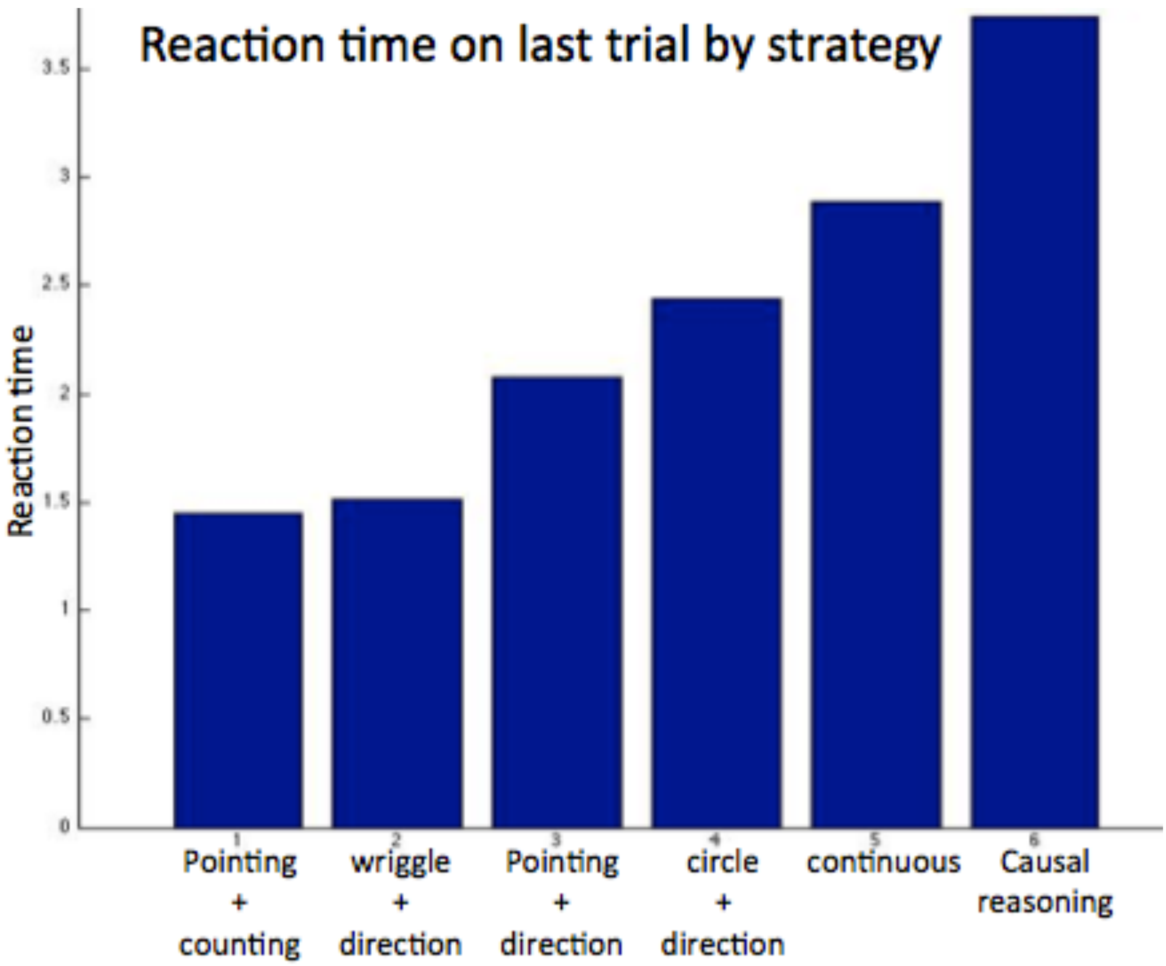
Flow chart diagram of typical developments



No very simple pattern of phase transition between two states is found, as suggested by Stephen et al 2009. Rather, a more complicated landscape of phase transition appears, with two attractors: the continuous curve-following and the discrete marking of the two directions, terminating in counting. Gestures accompanied by causal reasoning is generally instable as a strategy and will develop into either of the two stable states, the Continuous-silence and the Pointing-counting positions. Obviously, not all pairs of respondents reach one of these end points, but it seems as if those settling upon the less than optimal Continuous-silence strategy will typically remain there. All the different strategies characterized by a verbal indication of direction, however, may develop further to Pointing-direction or, ultimately, to the Pointing-counting strategy.

There are some indications that early behaviour during the trials may indicate later strategy choices. Early alternation between left-right movements might be indicative of later discretization in terms of Wriggle-direction of Pointing-counting, while, on the other hand, reliance upon causal arguments following wheel contours may be indicative of settling for the Continuity-silence strategy.

It is interesting to compare the strategies by reaction time at the last trial:



Causal reasoning is obviously the slowest strategy, followed by the more efficient continuous stragety. After this comes, in order of increasing efficiency, the three strategies involving verbal indication of wheel directions - the Circle, Pointing and Wriggling strategies with direction indications, respectively. It is remarkable that the Wriggle-direction strategy is almost as efficient as the Pointing-counting strategy, making it understandable that strategy developments may stop at the Wriggle-direction strategy.

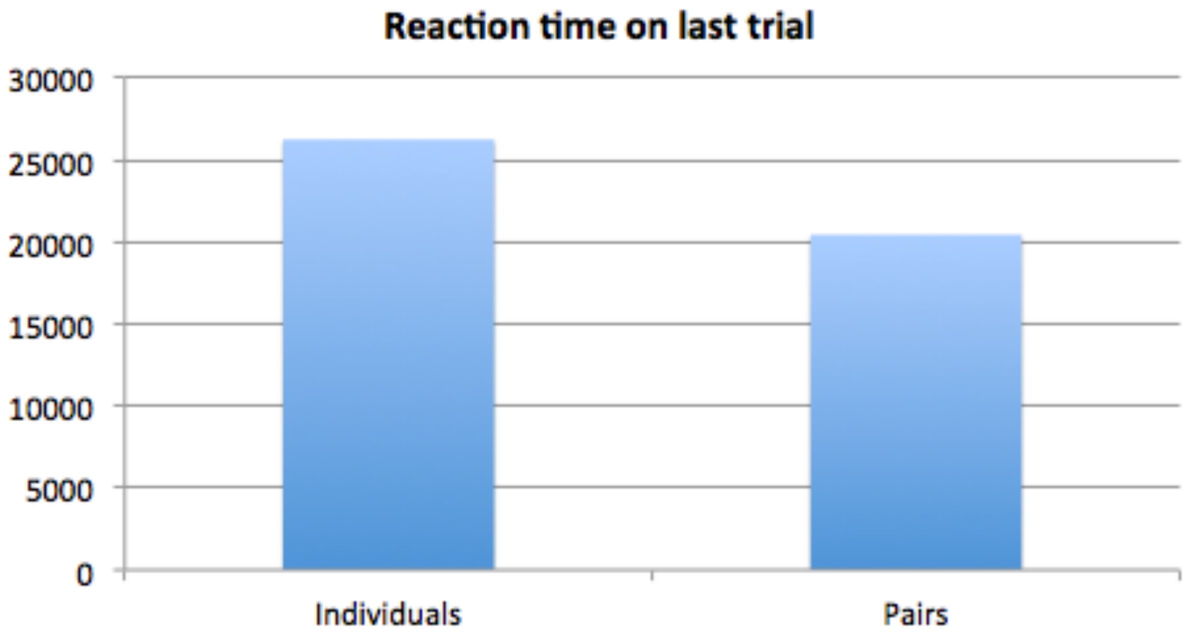
Conclusion

Embodiment and collaboration - two hypotheses

These results hold some important lessons as to the role of embodiment in diagram reasoning. It has often been pointed to the fact that diagrams facilitate reasoning by means of their spatial presentation of problems, making it possible to solve them by means of real or imagined bodily manipulation with parts of the diagram. Additionally, externalized diagrams on paper, board, screen or elsewhere facilitate the simultaneous or consecutive collaboration on the diagram by several persons. Both of these aspects, of course, are involved in the Cog Wheel experiment. As to the embodiment issue, an important result seems to be that "more" embodiment does not equal better solution strategy. The strategies which closely mimic the causal chain or the movement pattern of the wheels, touching the screen, are the least effective. The more effective strategies are the more abstract ones of Pointing-counting and Wriggle-direction which are by no means completely disembodied but which use gesture (pointing, wriggling) to address the discrete structure of the problem rather than its concrete, continuous materiality. We may consider the different gestures as diagrammatic representations of possible solution strategies. Here, the causal arguments contain, in a sense, too much information. This is also the case with the continuous curve which is more efficient than simple causal reasoning but remains a suboptimal solution. Getting to the optimal solution requires an instance of insight. This step of abstraction seems to involve moving focus from local information about how two wheels influence each other to a more abstract, global regularity about every second wheel moving in the same direction. To get to the optimal solution, then, it seems necessary to get away from a closely embodied interaction with the target. Indeed, certain solvers successfully used the wriggling-direction strategy at a distance of several meters from the screen.

This may give rise to the following hypothesis. Not all reasoning is based on immediate embodied experience – but rather on abstraction from immediate, concrete and local embodied experience. It should be added, however, that this abstraction is by no means completely disembodied but accompanied with specific embodied strategies supporting abstraction - in this case the wriggling and pointing gestures.

As to the collaboration issue, the provisional results of the Cog Wheel experiment seem to indicate that mixed-media strategies involving both language and gesture are the more efficient. Moreover, comparisons with individuals solving the same task seem to show that pairs are more efficient than individuals.[i]



Pilot data from Science in the City, pairs = 31 (n=62), individuals = 23

Why do pairs perform better? A hypothesis may be that several reasons combine here. One explanation is that the task affords abstraction - and as pairs speak, language may form a route to finding the more abstract solution strategies. Another explanation is that individuals may tend to get stuck with their first solution, while pairs may bring different perspectives and strategies motivating the intuition that there might be more than one solution strategy to be tried out. A further explanation may be feedback between the two parties, in the shape of one participant watching the other's gesture and drawing further conclusions; more generally in the shape of collaboration or competition or both (the two should not be seen as mutually exclusive, rather as two feedback aspects which may even enhance each other).

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[i] These provisional data were collected at a public performance at the "Science in the City" festival at the ESOF conference, Copenhagen, June 2014.