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Iconicity of Logic
- and the roots of the "iconicity" concept

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Abstract
It seems to be a standard assumption that Charles Morris originated the concept of "iconicity" on the basis of Peirce's icon-index-symbol discussion. This paper locates the origin in Peirce himself, in the context of judging the merits of different mathematical and logic representations – the more iconic such representations generally being preferable to less iconic ones, for scientific purposes. In Peirce's Collected Papers*, "iconicity" occurs in the discussion of different conventions in the logic representation system called "Existential Graphs". This paper provides the context of logic representations in order to show how Peirce's articulation of the concept of "iconicity" comes out of the attempt to find as iconic a way as possible to depict logical relations. Moreover, this indicates a use of "iconicity", from the very beginning, which addresses not only similarities between different visual representations – but also visual representations of abstract contents.*

In the context of "Iconicity in Language and Literature", it may seem odd to direct attention to the issue of *iconicity of logic*. Some may think that logic pertains to issues far from language and literature, others may suppose logic addresses structures which could never be characterized in terms of iconicity, cf. the phrase "symbolic logic".

I shall argue that not only is iconicity of logic relevant for language and literature, but the very notion of iconicity comes out of deep issues in the discussion of formal logic and different ways of representing it. If something exists which can be called logical structure, distinct from the different incarnations, representations, or formalizations of that structure, then those different formalizations may be measured by the degree to which they adequately depict logical structure. That is, by their degree of iconicity. This is not, however, a case of iconicity between two visual representations – rather, it

addresses the issue of visual representations of logical structure, which is not in itself visual.

This, again, raises important issues about the basic status of iconicity itself. Actually, the very *term* of iconicity emerges out of the discussion of different logic representations. There seems to be a widespread urban legend that the concept of iconicity originates in Charles Morris' interpretation of Peirce's icon-index-symbol triad, e.g., in his *Signs, Language and Behavior* (1946).[1] This, however, is not correct. The concept is already found in Peirce's doctrine of signs. And here, it is articulated exactly in the context of different logic representations. So, the very *origin* of "iconicity" lies in the interface between semiotics and logic, which I think might be a useful piece of knowledge, not least for the Iconicity research community.

In Peirce's *Collected Papers,* "iconicity" occurs only once, in a discussion of different possibilities for representing bound variables in his "Existential Graphs" notation of predicate logic: "But of what variety of Linear Continuity is the heavy line more especially the Icon in the System of Existential Graphs? In order to ascertain this, let us contrast the Iconicity of the line with that of the surface of the Phemic Sheet." (Ms. 300, "The Bedrock beneath Pragmaticism", 1905; 4.561n[2]). The quote addresses the use of so-called Identity Lines to refer to individuals – more about this below. There is not yet any occurrence of iconicity in the ongoing publication of the *Writings*, while one mathematical use is found in the *New Elements of Mathematics* selection of Peirce mss.[3] Further "iconicity" occurrences in the vast amount of unpublished Peirce manuscripts can, of course, not be precluded.

The converse, negative concept of "aniconicity" may also be found in the discussion of logic representations: "One system seems to be about as good as the other, except that unnaturalness and aniconicity haunt every part of the system of entitative graphs, which is a curious example of how late a development simplicity is." ("Logical Tracts no. 2", 1903, 4.434). Here, the discussion addresses Peirce's choice between two different graphical representations of Logic, his "Entitative" and "Existential Graphs", (cf. below). Also verbalization ("iconize") and adjectivizations ("iconic" and "iconical") of the term "icon" are widely used by Peirce, especially in the context of discussing logic representations. In this paper I shall investigate which ideas of logical iconicity are at

stake in Peirce's logic representations.

Peircean Iconicity

As is well known, Peircean iconicity is not restricted to visual nor perceptual similarity, nor to easily recognizable resemblance. Quite on the contrary, Peircean iconicity *began* with logic. At least two basic issues are addressed at length here: a) which parts or aspects of logic necessitate the use of iconic signs (cf. predicates rather than subjects)? – and b) the higher-level issue of which aspects of logic structure itself may be iconically expressed (cf. the choice between different representation systems)? Thus, the Peircean notion of iconicity goes far beyond perception, such as is evident from what I have earlier called his "non-trivial iconicity definition" (Stjernfelt 2007, 123f): "For a great distinguishing property of the icon is that by the direct observation of it other truths concerning its object can be discovered than those which suffice to determine its construction." ("That Categorical and Hypothetical Propositions are one in essence, with some connected matters", 1895, 2.279)

This surplus of information is accessed via manipulation of or experimentation with the icon – actions realizing deductive inferences. This implies that Peircean iconicity has a far wider extension than seen by many later iconicity scholars. The criterion of being an icon is simply whether such "other truths" may be inferred from it. Thus, the extension of iconicity not only comprises the ordinary series of pictures, images, and photographs, but also examples like:

- 2-D continuous charts

-> manipulation of icon: Finding routes on topographical maps, extrapolating graphs, etc.

- Algebraically expressed equations

-> manipulation of icon: solution of equations x +2 = 4 => x =2

- Aspects of linguistic syntax

-> manipulation of icon: "John beats Peter" => "Peter was beaten by John"

- and many more. Thus, Peircean iconicity addresses, from the outset, a logical issue: which information may be inferred from a sign?

Iconicity in Logic Formalizations

In order to understand the notion of logical iconicity, we must address the issue of how to express logical relations. Often, the logic tradition has favoured linguistic representations, like the syllogism "All men are mortal. Socrates is a man. Hence, Socrates is mortal". In the 19 C, however, increasing interest was given to graphical representations of logic, such as Euler Circles or Venn Diagrams. In the 1870s, F. A. Lange, in his *Logische Studien* (1877) argued that logic as such relies on spatiality and, hence, is best represented graphically - an argument which deeply impressed Peirce.[4] Famously, Frege in his 1879 *Begriffsschrift* was the first to introduce quantifiers and relational logic, simultaneously proposing a graphical formalization of propositional logic and first order predicate logic. Not knowing Frege's achievements, Peirce proposed alternative representations of the two, in 1880 and 1883-85, respectively. Twenty years later, he constructed an alternative graphical representation system, known as "Existential Graphs". Thus, Peirce constructed no less than two different sets of elaborated logic representations:

1) The "Algebra of Logic", formalizing propositional logic (1880) and first-order predicate logic (1885) in a linear language (which was the first version of the formal logic which, via Schröder, Peano and Russell, became modern standard usage)

2) The "Existential Graphs*"* (around 1900), formalizing propositional logic (*Alpha*), first-order predicate logic (*Beta*), and an unfinished series of further logics (*Gamma* – second-order Predicate Logic; modal logic; three-value logic; temporal logic; speech act logic, etc.)[5]

Algebra of Logic

Peirce's two 1880s logic representations appear in two homonymous papers:

1) “On the Algebra of Logic” 1880, concerning the formalization of Propositional Logic (“Logic of Non-relative Terms”)
2) “On the Algebra of Logic” 1885 – the introduction of quantifiers; the formalization of First Order Predicate Logic (“Relative Logic”)

Let us first take a look at his proposals for the connectives of Propositional logic (1880) – the very first version of modern formal logic.

	Peirce notation	Modern notation		
implication:	P[i] -< C[i]	$p \Rightarrow q$	$p \rightarrow q$	$p \supset q$
negation:	~B	$\neg p$		
and:	aXb	$p \wedge q$	$p \bullet q$	
or:	a+b	$p \vee q$		

Figure 1

As is evident, all of Peirce's proposals are syntactically equivalent to present-day use – the only difference lies in the shape of the individual sign of the connective. In some cases, there is even close relations among the signs’ actual character in Peirce and modern notation, respectively – the asymmetric, directed character of the implication sign, the prefix negation sign, the “and” signs taken from arithmetic multiplication.

The next step, the system for Relational logic, what is now called First Order Predicate Logic, followed in 1885 introducing quantifiers and polyadic relational predicates, just like Frege had done it six years earlier, but now in a linear, algebraic notation. Here, Peirce's proposal addressed which different aspects of relational logical expressions should be expressed iconically, indexically, and symbolically, respectively. If we take the sentence “Somebody loves something”, it will be expressed as follows:
$\Sigma_i \Sigma_j(l)_{ij}$ - meaning “There exists an i and there exists a j so that i loves j”. This expression now has three parts:

1) an index part – quantification – $\Sigma_i \Sigma_j$ – pointing out the objects to which the proposition refers

2) an iconic part – the Boolean part – describing the relations claimed to hold – $(l)_{ij}$ – "love" being a two-part relation iconically depicted by a bivalent predicate

3) a "token" (symbolic) part – presented by token signs which are conventional and general: *i; j; >;* Σ*; 0; l ...*

This apparatus allows the expression of existentially and universally quantified propositions:

$\Sigma_i \Sigma_j (l)_{ij} > 0$

– meaning: *something is a lover of something;* and

$\Pi_i \Sigma_j (l)_{ij} > 0$

– meaning*: everything is a lover of something*

Peirce's notation is algebraically motivated. Π_i means "For all *i* ..." while Σ_j means "There exists a *j* ...". Here, the Π_i notation has been chosen with reference to "Product" and Σ_j with reference to "Sum" – the Boolean idea being that the truth of a claim can be expressed by the value 1 and falsity by the value 0. Then, Π_i is the product of all the truth values of the single *i*'s. If the claim is false about just one *i* but true about all the others, then that single 0 suffices to make the whole product zero – meaning that it only holds for all *i*'s if the product is larger than zero. Conversely, Σ_j is the sum of all the truth values of the single *j*'s. If the claim is now false for all *j*'s except for one *j*, then this single 1 is sufficient to make the sum larger than zero – meaning there exists one *j* making the claim true.

Peirce quickly realized that the ">0" might be skipped as being superfluous as it appeared in all propositions and thus was pragmatically empty.

– Universal quantifier: $\Pi_x P_x$

– Existential quantifier: $\Sigma_x P_x$

$\Pi_x P_x$ (∏ for Product) - originally meaning the product of P_x cases > 0

– i.e., no single P_x case may be zero

$\Sigma_x P_x$ (Σ for Sum) meaning the sum of P_x cases > 0

– i.e., at least one single P_x case must be > 0

Figure 2

Let us compare this representation system to Frege's 1879 notation and modern notation – all of the three here expressing the proposition that "There exists a red ball":

- Frege 1879

- Peirce 1870-1885 $\Sigma_x(ball_x \bullet red_x)$

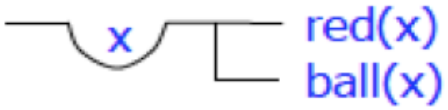
- Peano 1900, Russell 1910, & later $\exists x\,(ball(x) \wedge red(x))$

Figure. 3

We shall not here go deeply into the details of Frege's more cumbersome notation, suffice it to point out the close relatedness between Peirce's notation and modern notation developed on the basis of it. Peano took it over from Schröder, substituting the inverted Es and As (for "exist" and "all") for Peirce's algebraic notions, but the overall syntax remained unaltered.

Similarly: “All balls are red”

- Frege 1879



- Peirce 1870-1885

$\Pi_x(ball_x —< red_x)$

- Modern notation – Peano 1900, Russell 1910 & later

$\forall x\ (ball(x) \supset red(x))$

Fig. 4

Existential Graphs

A chief occupation of the mature Peirce after the turn of the century was the construction of a quite different logic formalism, which he baptized as “Existential Graphs”.[6] An important question here immediately jumps to mind: Why “Existential Graphs”? Peirce’s 1880-85 notation was the origin of modern formal logic via Schröder, Peano, and Russell (Frege's *Begriffsschrift* notation never came in use) and had received some degree of recognition. So why did Peirce develop an alternative notation 20 years later (from approximately 1897 onwards)? He had every reason to remain satisfied with his 1880-85 achievements, which were even spreading to Europe via the work of Schröder. Peirce could not know that his role in the origin of modern formal logic would soon be forgotten along the Schröder-Peano-Russell line with the result that most later logicians have no knowledge of his role and, in many cases, even erroneously think that Frege was responsible rather than Peirce. This dispute about priority hence did not occupy him and has only become an issue addressed by (much) later intellectual historians (cf. Putnam, Anellis, etc.).

So why did Peirce set out to begin from scratch, constructing a wholly new graphical representation system for Propositional and First Order Logic? *For reasons of iconicity!*

As Peirce quite explicitly states: The purpose of “Existential Graphs” is “... to

afford a method (1) as ***simple*** as possible (that is to say, with as small a number of arbitrary conventions as possible), for representing propositions (2) as ***iconically***, or diagrammatically and (3) as ***analytically*** as possible." (1905; Ms. 300, 4.561n)

What, then, is the iconicity claimed for these graphs? Let us take a look at the simplest "Existential Graphs", the Alpha system formalizing, again, Propositional Logic:

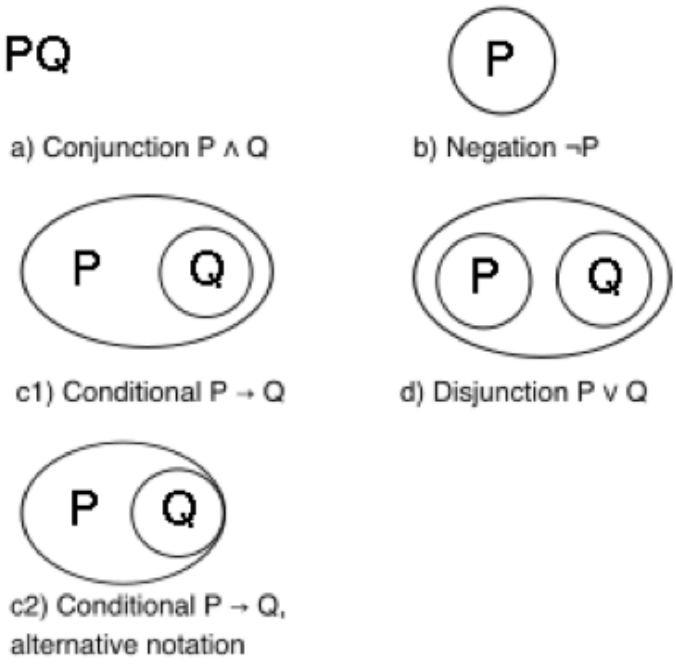


Figure 5

Alpha Graphs are much simpler than the 1880 Algebra of Logic. It has two primitives only:[7]

- *Co-localization: "And" - the location of two propositions at the same part of the page means the conjunction of the two*
- *Inclusion: "Negation" - the inclusion of a proposition in a "cut", separating it from the rest of the page means the negation of that proposition*

Behind these two conventions lies the interpretation of the blank page. It is called the "Sheet of Assertion" or the "Phemic Sheet" and is taken to iconically depict the Universe

of Discourse which is what the whole set of possible propositions at stake refers to. Thus, the continuous, empty page refers, implicitly, to all relevant truths.

Peirce's idea is now that this is a *more iconic way* of representing logical relations: to represent two propositions side by side, embraced in the same, true part of the universe, is a more iconic representation for "and" than representing them with an additional sign like "×" or " ∧ ". And to represent negation by *cutting away* the proposition from the sheet of truths is considered more iconic than attaching a purely conventional negation sign like "~" or "¬" to that proposition.

Among the advantages Peirce saw in Existential Graphs was their multiple interpretability:

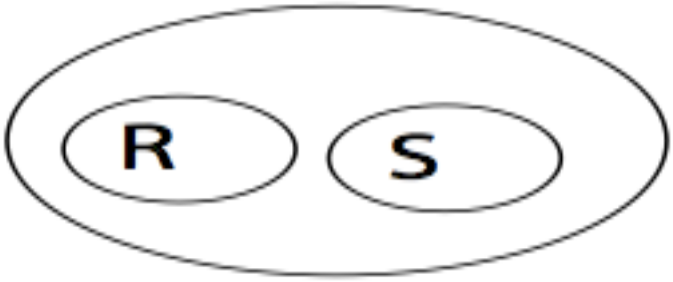


Figure 6

The non-linear Existential Graphs are multiply interpretable and may be read as realizing a series of logical propositions which, in the ordinary notation, would require proofs to establish as equivalent. Thus, the graph depicted, in ordinary language reading "It is not the case that not-R and not-S" is immediately equivalent to the following compound propositions:

- ¬ (¬R ∧ ¬S)
- ¬S ⇒ R
- ¬R ⇒ S
- R ∨ S

...

This amount of different depictions, in linear logic (including Peirce's own earlier Algebra of Logic), of the very same structure is taken to be an anti-iconic property which should be avoided.

Beta Graphs

The next step was to substitute Beta Graphs for the Algebra of Logic notation of First Order Predicate Logic. It builds on Alpha graphs, and adds to Alpha Conventions further ideas:

- Dots refer to individual objects
- Lines of Identity connecting two dots identify those individuals - thus each Line of Identity refers to one individual or variable
- Lines of Identity may be composed into Ligatures, structures of Lines each referring to the same constant or variable - Ligatures may thus identify several variables, just as they may cross negation cuts
- Lines of Identity may connect to hooks of Predicates written directly on the Sheet so that *S*————*blue* means that the individual S has the property blue
- Polyadic predicates have as many hooks as their valency indicates, and they may be connected to a Line of Identity at each hook, thus *A*——*loves*——*B* means "A loves B" and ——*loves*—— means "Somebody loves something". Thus a basic iconicity holds between the number of relata of a relation and the amount of hooks of the corresponding predicate.
- Thus, the Line of Identity also expresses quantification; the outermost end of an Identity line signifies quantification. If the line ends directly on the sheet or is enclosed by an even number of negations, this means existential quantification, "There exists an individual ..."; if the line is oddly enclosed by negations, this means negative universal quantification, "It is not the case that there exists an individual ..." - that is, "No x ..."

Thus, the Line of Identity takes care of identity, existence, predication as well as subsumption - the four different functions of the copula which, in the Frege-Russell tradition, was analyzed as ambiguous for that very reason.

Let us take a couple of examples of Beta Graphs to see the expression of quantification and variables by means of Lines of Identity:

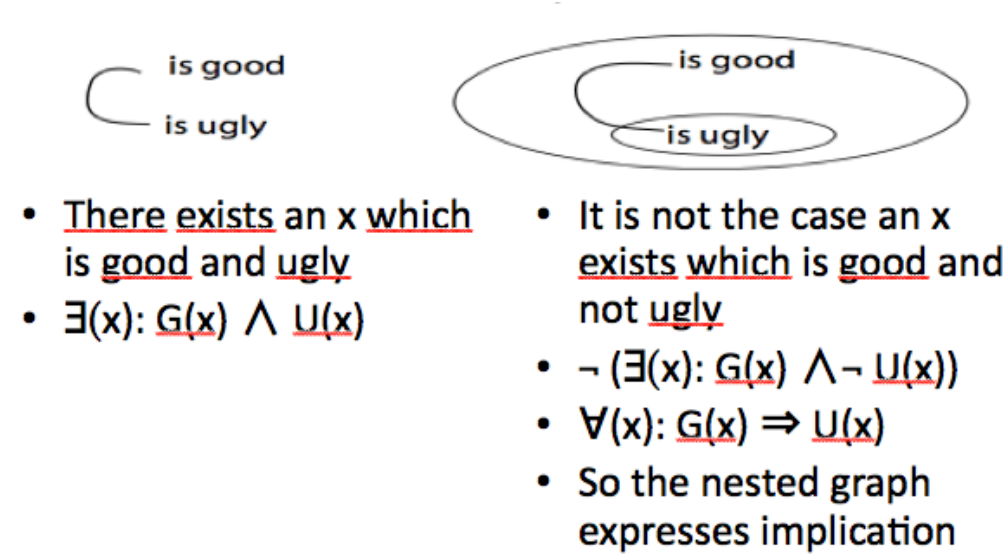


Figure 7

The first graph has both ends on the sheet and is thus quantified existentially, meaning: there exists an *x*. This line is connected to the hooks of two one-slot predicates, "is good" and "is ugly", respectively. The whole graph hence means "Something exists which is good and ugly". The standard formalization is given for comparison. In the second graph, the Identity Line has its outermost end in a negative area. Thus, existence is denied: "It is not the case an x exists which is good and not ugly". This, of course, is equivalent with the sentence: "If anything is good, it is ugly" – or, again, "For all *x*s, if *x* is good, it is ugly", or colloquially, "All which is good is ugly". Thus, the nested graph of two cuts, one within the other, just like in Alpha Graphs, expresses material implication, *if-then*. Peirce took this as a particularly successful iconic representation, showing how the consequence is contained in the premises.

Lines of Identity

In case of polyadic predicates, several Lines of Identity may be attached to different slots in the predicate:

Figure 8

"If there exists anyone, he blames somebody to somebody else" – or "Everybody blames someone to somebody". It is easy to see that the number of individuals referred to by such a proposition is equivalent to the number of Lines of Identity – in this case, three. According to Peirce, Lines of Identity and their composition into Ligatures are, for this reason, *more iconic*, because of this simple rule: *there is one Ligature for one variable* – contrary to the linear notation where the same variable is repeated several times through the formula. Compare these two representations of the same proposition:

$\forall\ (x)$: G(x) $\Rightarrow$ U(x)

Figure 9

In the linear notation, the variable *x* appears *thrice* – in the EG notation, the same variable appears only as *one* Ligature. This is *more iconic*, as there *is* only one variable referred to, not three.

Peirce first experimented, in the 1890s, with a dual system called "Entitative Graphs", where the end point of an Identity Line on the Sheet meant *Universal* instead of Existential Quantification and where the fundamental connective, represented by co-localization, was OR instead of AND.

Giving up "Entitative Graphs", he argued that it was *more iconic* for a simple end point of a line to mean "There exists an *x* ..." than to mean "For all *x*s ..." – and it was *more iconic* that "PQ" meant "P and Q" rather than "P or Q". The reason behind these iconicity claims is that *one* point iconically presents the idea in Existential Quantification that (at least) *one* x exists. The quote referred to in the quote on "aniconicity" above rejects "Entitative Graphs" exactly because of their lack of iconicity.

Iconicity in EGs vs. Linear Notation

Let us now sum up the arguments for Existential Graphs being more iconic than the linear notations of the Algebras of Logic:

1) *The blank sheet,* as mentioned, is an iconic sign for the Universe of Discourse because it involves all possible points – that is, all possible true existential propositions;
2*) Co-localization* as a sign for "and" is more iconic than "p∧q" or "p•q" because it immediately gives the idea that the two propositions joined form parts of the same Universe of Discourse;
3) The *cut* (or "sep") as a sign for negation is more iconic than *~p,* or *non-p,* or ¬*p* because it literally separates the negated from the universe of discourse;
4) The *Line of Identity* as a sign for existence, identity, subsumption, predication, all at once, is more iconic than the various means used in the Algebra of Logic. As to quantification, it is more iconic than the algebraic Product/Sum quantifier symbols and their repeated *x*'s because of its unity and its continuity. As to identity, it is more iconic than conventional signs like "=" because of its continuity, directly connecting the two points identified. As to predication, it is more iconic than P(*x,y*), again because of its continuity, directly connecting the variable with the relevant slot in the predicate.

As noted above, Peirce compared Ligatures combining Identity Lines with another device called "Selectives". What lead him to this consideration was that in cases with many Identity Lines and Ligatures, some of them crossing one another, such lines may form a maze which is not immediately perspicuous to the observer. In such cases, a Line of Identity may be cut into pieces, and each piece then identified by an attached letter instructing the observer that the line pieces with the same letter should be read as referring to the same individual or variable. Take the below graph, meaning "There exists a woman and if she has a child, she loves it". The first version involves crossing Lines of Identity, necessitating the convention of a small "bridge" preventing the two from merging (which would indicate reference to the same individual). The second version shows how Selectives reintroduce the array of X's and Y's from the linear notation:

Figure 9

Here, Peirce definitely preferred Ligatures – and the very quote in which he introduces the notion of "iconicity" occurs in an argument addressing exactly this: that Ligatures are more iconic than Selectives because of their continuity. He goes on to compare the explicit reference of 1-D Ligatures involving individual variables to the 2-D continuity of the whole sheet which then refers to the whole Universe of Discourse to which no particular attention is paid but which has the objects highlighted by the Ligature as parts.

5) The *end point of line* as the sign for Existential Quantification is more iconic than "Σ_x" or "$\exists$ x" because it selects *one* point from the Universe of Discourse;

6) The negated Existential Quantification is an iconic sign for Universal Quantifier, because Universal claims are inherently negative – to claim that All Xs are Ys is to say

there are No Xs that are not Ys – so to actually find such an X provides a counterexample;

7) Predicates are represented with hooks (empty slots) whose number iconically corresponds to the valency of the predicate;

8) The nested structure of negations gives, at the same time, the scope of quantifications;

9) In implications, the implied is in an inner cut of the outer cut – im-plication.[8]

Now we are in a position to appreciate the early appearance of the notion of *iconicity* in the middle of an argument pertaining to Existential Graphs. This addresses the lack of iconicity in Selectives mentioned in bullet 5), arguing for the superiority of the Identity Line notation instead. Peirce takes the deficits of Selectives one by one - first they are not as simple; second they are not as iconic, and third, they are not as analytical (the "two S's" here correspond to the X's in the Figure 9 example above):

> "The first respect in which Selectives are not as analytical as they might be, and therefore ought to be, is in representing identity. The identity of the two S's above is only symbolically expressed (...) Iconically, they appear to be merely coexistent; but by the special convention they are interpreted as identical, though identity is not a matter of interpretation -- that is of logical depth -- but is an assertion of unity of Object, that is, is an assertion regarding logical breadth. The two S's are instances of one symbol, and that of so peculiar a kind that they are interpreted as ***signifying***, and not merely ***denoting***, one individual. There is no analysis of identity. The suggestion, at least, is, quite decidedly, that identity is a simple relation. But the line of identity which may be substituted for the selectives very explicitly represents Identity to belong to the genus Continuity and to the species Linear Continuity. But what variety of Linear Continuity is the heavy line, more especially, the Icon in the System of Existential Graphs? In order to ascertain this, let us contrast the Iconicity of the line with that of the surface of the Phemic Sheet. The continuity of this surface being two dimensional, and so polyadic, should represent an external continuity, and especially, a continuity of experiential appearance. Moreover, the Phemic Sheet iconizes the Universe of Discourse, since it more immediately represents a field of Thought, or Mental Experience, which is itself directed to the Universe of Discourse, and considered as a sign, denotes that Universe. Moreover, it [is because it must be understood] ***as*** being directed to that Universe, that it is iconized by the Phemic Sheet. So, on the principle that logicians call "the ***Nota notae***" that the sign of anything, X, is itself a sign of the very same X, the Phemic Sheet, in representing the field of attention, represents the general object of that

> attention, the Universe of Discourse. This being the case, the continuity of the Phemic Sheet in those places, where, nothing being scribed, no ***particular*** attention is paid, is the most appropriate Icon possible of the continuity of the Universe of Discourse -- where it only receives ***general*** attention as that Universe -- that is to say of the continuity in experiential appearance of the Universe, relatively to any objects represented as belonging to it. / (...) Now for the continuity of the line of identity. This being one-dimensional, or dyadic, (i.e. running two ways only,) should represent an internal, or mental, continuity; and being definitely marked, should iconize a continuity of attention. But the heavy line is generated by the continuity of the different places of a heavy dot, which is the appropriate icon of an individual object in a Universe of continuous co-being; and, therefore, the continuity of the line is, best, the Icon of the continuity in attentive observation of an individual object." (Ms. 300, "The Bedrock beneath Pragmaticism", 1905; partially in 4.561n)

The notion of "iconicity" occurs in the comparison of the continuity of the Line of Identity with that of the whole Sheet of Assertion on which the Line is drawn. The Sheet is two-dimensional and objective in the sense that it represents the entire world, which the actual piece of reasoning addresses. The Line is one-dimensional only and makes explicit a small part of the former. It is "mental" not in the sense of psychological but in the sense of representing the continuous existence of the object it refers to, which is granted by an act of continuous attention to that object. In that sense, it is what Peirce elsewhere calls "the immediate object" of a sign – which is the indexical connection claimed to exist between the sign and the object, granted (when it actually does exist) by the simultaneous existence of that object and attention to that object. Thus, this was what this first - as far as we know - first occurrence of the term "iconicity" was intended to explain.[9]

The demand for as high a degree of iconicity as possible, however, does not imply the suppression of indexicality or symbolicity. Peirce, proud of his invention of the Line of Identity, rather claims that the advantages of this particular notation lies in its satisfying an ideal of *equilibrium* between these three sign functions:

> "The value of an icon consists in its exhibiting the features of a state of things regarded as if it were purely imaginary. The value of an index is that it assures us of positive fact. The value of a symbol is that it serves to make thought and conduct rational and enables us to predict the future. It is frequently desirable that a representamen should exercise one of those

> three functions to the exclusion of the other two, or two of them to the exclusion of the third; but the most perfect of signs are those in which the iconic, indicative, and symbolic characters are blended as equally as possible. Of this sort of signs the line of identity is an interesting example. As a conventional sign, it is a symbol; and the symbolic character, when present in a sign, is of its nature predominant over the others. The line of identity is not, however, arbitrarily conventional nor purely conventional. Consider any portion of it taken arbitrarily (with certain possible exceptions shortly to be considered) and it is an ordinary graph for which Fig. 81 might perfectly well be substituted. But when we consider the
>
> —is identical with—
>
> Fig. 81
>
> connexion of this portion with a next adjacent portion, although the two together make up the same graph, yet the identification of the something, to which the hook of the one refers, with the something, to which the hook of the other refers, is beyond the power of any graph to effect, since a graph, as a symbol, is of the nature of a ***law***, and is therefore general, while here there must be an identification of individuals. This identification is effected not by the pure symbol, but by its ***replica*** which is a thing. The termination of one portion and the beginning of the next portion denote the same individual by virtue of a factual connexion, and that the closest possible; for both are points, and they are one and the same point. In this respect, therefore, the line of identity is of the nature of an index. To be sure, this does not affect the ordinary parts of a line of identity, but so soon as it is even ***conceived***, [it is conceived] as composed of two portions, and it is only the factual junction of the replicas of these portions that makes them refer to the same individual. The line of identity is, moreover, in the highest degree iconic. For it appears as nothing but a continuum of dots, and the fact of the identity of a thing, seen under two aspects, consists merely in the continuity of being in passing from one apparition to another. Thus uniting, as the line of identity does, the natures of symbol, index, and icon, it is fitted for playing an extraordinary part in this system of representation." ("Logical Tracts, N° 2", 1903, 4.448)

Peirce's celebration of the harmonious concert of symbol-index-icon begins "from above", as it were. The Line of Identity is a symbol, because it rests on a convention, the convention discussed above giving the rules according to which it may express identity, existence, predication and subsumption, all at the same time. But these conventions make rules for a sign which is already fit to serve these purposes. Its indexicality here is argued by observing that, as all general signs, it may only exert its general, symbolic function in

the shape of an actual sign token existing here-and-now – a line drawn on a sheet, in this case. And this line *factually* connects its extremities – unlike other candidates for the same general meaning. This factual connection, again, is supported on the most basic level, by iconicity – by the continuity of the Line of Identity depicting the continuity of existence of the constant or variable referred to.

Conclusion

The birth of "iconicity" takes place in the middle of an investigation aiming to settle which logic representation most iconically represents logical structure. Let us sum up the arguments schematically:

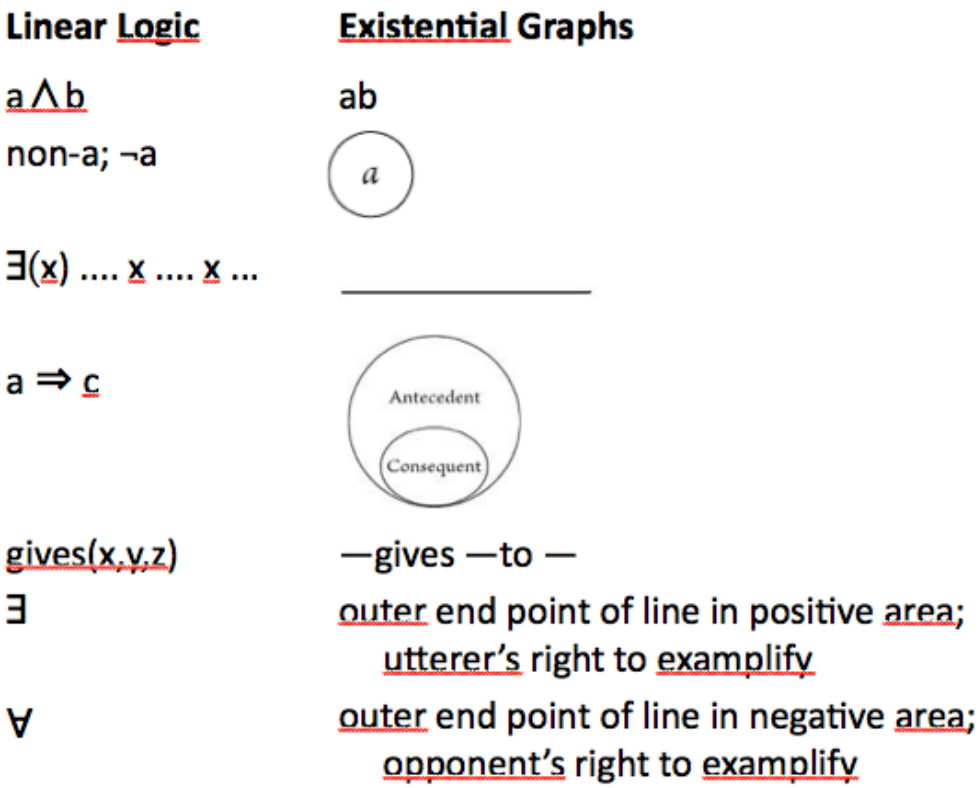


Figure 10

The EGs were taken by Peirce as a means of analyzing logical structures as unanimously and detailed as possible – not as an easy calculus aiming at computing quick inference results. It is for this reason he is so adamant in his pursuit of iconicity. A calculus – with an eye to quick reasoning – may, quite on the contrary, benefit from being *less* iconical – having more logical primitives, more ways to express the same thing, more rules of thumb for shortcuts.

But Peirce's obstinate demand for iconicity in logic also has relevance for iconicity in general. It argues the more general case that different spatial representations of abstract structures – in this case, topological representations of logic – may be subject to iconicity criteria. It argues that iconicity plays a basic role in the selection among competing scientific formalizations – also in abstract and formal sciences. And it argues that there may be a trade-off between optimal iconicity and heuristic utility, which may strike different compromises depending on pragmatic purposes.

Abstract structures also inhere in more immediately accessible iconic signs like paintings, photos, movies, diagrams, poetry etc. so we should expect to find the different degrees and modes of logical iconicity playing a role in such representations as well.

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[1] For example: "The concept of iconicity was first proposed by Morris", (Nöth 1990, 123). Also the idea of "degrees of iconicity", so central to Peirce's introduction of the notion, is routinely ascribed to Morris 1946. A similar urban legend seems to pertain to Peirce's notion of "aniconicity": "The concept of aniconicity was first introduced by T. A. Sebeok (1979) as the “complementary obverse” of iconicity" (Sonesson 1999). In both cases, we may add, the "concepts" referred to may have been discussed long before Peirce, *Kratylos* coming to mind as an early example. It was Peirce, however, not Morris or Sebeok, who introduced the notions of "iconicity" and "aniconicity" to address those concepts.

[2] References to Peirce's *Collected Papers* follow standard practice: volume number and paragraph, "4.561n" meaning a note to volume 4, paragraph 561.

[3] An even earlier use of "iconicity" is found in Ms. 229 (NEM II, 595), from around 1897 (personal comm. André de Tienne), a small text on the "logic of number" where Peirce discusses his definition of mathematics as the science that studies hypotheses. Here, he addresses the idea of the contemporaneous Scottish mathematician George Chrystal that mathematics is defined by the "definiteness" of mathematical conceptions and their "finite number of specifications". Peirce remarks upon the low degree of definiteness of topology, likening it to other, non-mathematical conceptions which may also display degrees of definiteness. This leads him to the remark: "I incline to suspect that Prof. Chrystal has confounded definiteness with iconicity, or the palpability of being represented in a diagram." In Peirce's philosophy of mathematics, diagram tokens form the access to diagram types, incarnating selected mathematical properties. Thus, the iconicity of diagrams is what makes possible knowledge about mathematical objects. Thus, the use of "iconicity" here addresses mathematical representation devices, just like the CP use addresses logical representation systems.

[4] Cf. Bellucci 2013.

[5] Actually, Peirce constructed, in the 1890s, a third system of so-called "Entitative Graphs" forming a dual system as compared to Existential Graphs, soon concentrating on the latter, cf. below.

[6] Much interesting scholarship has emerged investigating Existential Graphs in recent decades - see references to Roberts, Zeman, Shin, Pietarinen, Dau, etc., and the special issue of *Semiotica* ”Diagrammatical reasoning and Peircean Logic Representations” (Queiroz & Stjernfelt 2011).

[7] Peirce was the first to realize that all logical connectives could be defined in terms of one sign only, that for "neither-nor" (later called "Sheffer's Stroke" after the logician who rediscovered it) - for perspicuity, however, he preferred the EG Alpha version with two connectives - maybe because it does not seem easy to come up with a simple iconic sign for "neither-nor".

[8] You may add a futher feature of iconicity not implied directly by the Alpha and Beta conventions, which is the stepwise structure of logical proofs as an iconic sign of the dialogic structure of logic (see Pietarinen (2006) and further refs.). Thus, Peirce takes this stepwise structure as referring to the alternate efforts by an utterer and his opponent – in Peirce, a Graphist and a Grapheus – taking turns in making changes on the Existential Graph, one trying to prove, the other to disprove it. Connected to this idea is his reinterpretation of Existential and Universal Quantifiers as the right to select an instantiation by the defender, resp. the attacker of a proposition.

[9] As mentioned above (n3), there is an earlier (1897) occurrence of the term, but the 1905 occurrence is the first one supported by a thorough discussion of iconicity and degrees of iconicity of different representation devices.