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Moving Pictures of Thought

Diagrams as Centerpiece of a Peircean Epistemology

Recent developments in semiotics, semantics, and linguistics tend to give concepts like “schema,” “frame”, “gestalt,” and the like a renaissance in the description of signification processes. The actual cognitive semantics tradition (Lakoff, Johnson, Talmy, Turner, Fauconnier, etc.), for instance, highlights the central role of schemata and their mappings between conceptual spaces in the description of many levels in linguistics. Another related development is the renewed interest in diagrammatic calculi in the computer science and AI communities, documented in e.g. the influential *Diagrammatic Reasoning* volume (Glasgow 1995) - where the diagram category, however, is most often taken for its common sense value as an opposition to the symbol category; little effort is spent on determining the general status of the diagram as such.

This return of schematic iconicity in semiotics is probably the main event in semiotic scholarship during the recent decades, but it has not, until now, received a proper meta-theoretical treatment making clear the very concept of schema itself. This is a strange fact; in Peirce we find drafts for precisely such a theory in his general observations on the concept of *diagram*. While Peirce’s systems for logic diagrams (his *alpha*-, *beta*-, and *gamma*-graphs implementing propositional logic, first-order-predicate logic, and various types of modal logic and speech act logic, respectively[i]) have received considerable attention in recent years because of their indication that iconic representations of logic are possible and even to some extent heuristically superior to symbolic logic systems, Peirce’s general notion of diagram has passed much more unnoticed. This might be for editorial reasons - Peirce’s central arguments concerning the general diagram category are not to be found in the *Collected Papers* - but still the diagram concept plays a central, not to say *the* central, role in the mature Peirce's semiotics. In particular "PAP", a paper from 1906 (Robin 293, published in NEM IV), makes clear the crucial part played by the diagram and diagrammatic reasoning in Peirce. The present schema and diagram research would no doubt benefit from the knowledge of Peirce's general diagrammatic philosophy.

The aim in this chapter is twofold: firstly, to present and discuss Peirce's general diagram concept and its central role in his semiotics and in his philosophy as a whole, and, secondly, to argue for the significance, beyond Peirce philology, of this diagram concept for semiotics and epistemology of our day.

The Diagram as Icon

The diagram is an icon. In the taxonomy of signs, thus, the diagram form the second subcategory among the three types of hypoicons[ii] - images, diagrams, and metaphors, respectively ("Syllabus" 1903, EPII 274; 2.277) - even if Peirce elsewhere notes that sharp distinctions among icons are not possible due to the inherent vagueness of the concept. Being an icon, the diagram is characterized by its similarity to its object - but while the image represents its object through simple qualities and the metaphor represents it through a similarity found in something else, the diagram represents it through a skeleton-like sketch of relations (mostly dyadic, apparently in an attempt at justifying the three icon subtypes triadically). Knowing the inclusive nature of Peirce's triads in general, it follows that non-degenerate diagrams will include images, while non-degenerate metaphors will contain diagrams (and images).[iii] Still, this tripartition of icons is easy to overlook as yet another detail in the tree of ever trifurcating triads in Peirce's architectonic; it does not reveal the crucial role played by diagrams in Peirce's epistemology. To grasp this, a further investigation of the very definition of the icon is necessary.

The Non-trivial Icon Definition

The icon, of course, is defined as the sign referring to its object by virtue of similarity. Now, Peirce himself admitted the deliberate vagueness of this definition: an icon may refer to any object possessing the qualitites in question – and as discussed in the previous chapter, a strong tradition in 20C philosophy has attacked such definitions for being so vague as to be completely meaningless. The dangers in the similarity concept are many: the trivializing of it to identity; the psychologizing of it to refer to subjective feelings or judgments of resemblance; the lack of criteria for judging two phenomena similar. These traditional drawbacks of similarity are overcome by Peirce's non-trivial because *operational* account of similarity. In 1895, it is stated as follows: "For a great distinguishing property of the icon is that by the direct observation of it other truths concerning its object can be discovered than those which suffice to determine its construction" ("That Categorical and Hypothetical Propositions are one in essence, with some connected matters," 2.279). This epistemologically crucial property of the icon is nothing but an operational elaboration on the concept of similarity. The icon is not only the only kind of sign involving a direct presentation of qualities pertaining to its object; it is also - and this amounts to the same - the only sign by the contemplation of which more can be learnt than lies in the directions for its construction. This definition separates the icon from any psychologism: it does not matter whether sign and object for a first (or second) glance seems or are experienced as similar; the decisive test for its iconicity rests in whether it is possible to manipulate the sign so that new information as to its object appears. This definition is non-trivial because it avoids the circularity threat in most definitions of similarity. At the same time, it connects the concept of icon intimately to the that of deduction. This is because in order to discover these initially unknown pieces of information about the object hidden in the icon, some deductive experiment on the icon must be performed. The prototypical icon deduction is the manipulation of a geometrical figure in order to observe a theorem - but the idea is quite general: an icon is characterized by containing implicit information which in order to appear must be made explicit by some more or less complicated procedure accompanied by observation. As early as 1885, Peirce writes ("On The Algebra of Logic"), discussing the syllogism, but with evident implications for the icon category as a whole, that "... all deductive reasoning, even simple syllogism, involves an element of observation; namely, deduction consists in constructing an icon or diagram the relations of whose parts shall present a complete analogy with those of the parts of the object of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts" (W5, 164; 3.363).[iv] This property clearly distinguishes it from pure indices and symbols: If we imagine a pure, icon-less index (only possible as a limit case), then it would have a character completely deprived of any quality, a pure here-now of mere insistence, about which we would never be able to learn anything further, except exactly by some kind of icon of it. And if we imagine a purely symbolic sign (also a limit case), say e.g. the variable x, we could not learn anything about it except when placing it in some relation, syntax, system, context or other, that is, in some kind of iconical relationship. From this operational icon definition, connection lines run to a bundle of Peircean themes: the abductive guess as the suggestion of an icon as a general answer covering the particuar question present; icons as providing the predicative, descriptive side of any signification process; the pragmatic maxim's conditional definition of concepts described by an icon showing which operations we could conceivably perform on an object subsumed under the concept; the scientific community's unlimited semiosis converging towards truth, that is, an ever more elaborate icon possessing still more operational possibilities. We shall touch upon some of these issues later during the discussion of the type of icon making all this possible: the diagram.

The Operational Criterion and the Extension of the Icon Category

It is a well-known fact that Peirce's icon definition sets it apart from spontaneous tendencies to privilege visual icons. It is a more controversial fact that the operational icon definition extends the icon category considerably, measured against the spontaneous everyday conception of resemblance. Peirce's logic graph systems as iconic calculi already indicates this change: they demonstrate that systems normally considered symbolic possess an ineradicable iconicity.[v] Using Peirce's sign concepts, namely, it is no longer possible to speak about iconicity and symbolicity as two concurrent modes of representation of the same content: if the same logical calculus may be represented in two ways, this indicates that the "symbolic" representation did, in fact, already possess an iconic content: the possibility of experimentation on the calculus resulting in new insight grants - due to the operational icon criterion - that it is in fact an iconic calculus.[vi] Thus, when the operational criterion is adopted, icons become everything that can be manipulated in order to reveal more information about its object, and algebra, syntax, formalizations of all kinds must be recognized as icons; in the "That Categorical and Hypothetical Propositions ...", Peirce adds that these types of signs are even icons par excellence due to their capacity for revealing unexpected truths: “Given a conventional or other general sign of an object, to deduce any other truth than that which it explicitly signifies, it is necessary, in all cases, to replace that sign by an icon. This capacity of revealing unexpected truth is precisely that wherein the utility of algebraical formulae consists, so that the iconic character is the prevailing one” (1895, 2.279). This, in turn, implies that we, in the operational icon definition, find a useful criterion to distinguish fertile from less fertile formalization: the good formalization is one which permits manipulation in order to reveal new truths about its object; formalizations which only permit this to a small extent or not at all may be discarded.[vii]

The Diagram’s Status in Iconicity

Given the operational icon criterion, we are now able to appreciate the central role played by diagrams in the icon category as such. As soon as an icon is contemplated as a whole consisting of interrelated parts whose relations are subject to experimental change, we are operating on a diagram. Thus, the inclusion of algebra, syntax, and the like in the icon category takes place thanks to their diagrammatic properties - but the same goes for your average landscape painting as soon as you stop considering its simple qualities, colors, forms etc. and move on to consider the relations between any of these parts and aspects. As soon as you judge, for instance, fore-, middle-, and background and estimate the distance between objects depicted in the pictorial scene, or as soon as you imagine yourself wandering along the path into the landscape, you are operating on the icon - but doing so in this way is possible only by regarding it as a diagram. You may have no explicit awareness,[viii] it is true, of the rules which permits you to follow the imaginary track (the laws of perspective permitting you to construct the scene, gravity keeping you on the ground etc.), but still they are presupposed due to the organization of your perception apparatus[ix] and your tacit common sense knowledge. The principles *could* be made explicit, and this is what counts. Thus, it is hard to take a closer look at any icon without at least performing proto-diagrammatic experiments with it to reveal some of the implicit truths inherent therein. Thus, the use of a sign as a pure image is more like a limit case as when you enjoy the overall impression or *Stimmung* of a painting without going into any details. On the other hand, the appreciation of a metaphor may seem automatic, but recent metaphor research supports what lies implicit in Peirce’s thought: that a diagrammatic analysis - be it conscious or not - precedes any metaphor consisting in the recognition of diagrammatic schemas in one phenomenon which may be used in understanding another. The metaphor of an ‘ancestral tree’ thus presupposes that the formal branching diagram is mapped from a tree onto family structure. Far from all metaphorical mappings are so easy, of course, but it seems reasonable to assume that the mapping of diagrammatic structure between conceptual spaces plays a central role in metaphor in general.[x] Thus, the diagrammatic way of interpreting an icon seems central as soon as any part of the internal mereological structure of the icon is taken into consideration. The diagram’s skeleton-like, relational, and highly stylized picture of its object is at stake also when clothed in simple image qualities and hidden in the metaphors’ import reference to other empirical phenomena.

Now, let us look closer at how Peirce dissects the single elements and phases in the diagrammatic interpretation process. As already mentioned, one essay stands out when it comes to detailed analysis of this process, namely one of the drafts for “Prolegomena to an Apology for Pragmaticism” from 1906. The paper in question is Robin catalogue number 293 and is also known as “PAP” from Peirce’s own abbreviation. The next passage will present the central quote from that paper, describing the diagrammatic interpretation process *in extenso*.

> To begin with, then, a Diagram is an Icon of a set of rationally related objects. By *rationally* related, I mean that there is between them, not merely one of those relations which we know by experience, but know not how to comprehend, but one of those relations which anybody who reasons at all must have an inward acquaintance with. This is not a sufficient definition, but just now I will go no further, except that I will say that the Diagram not only represents the related correlates, but also, and much more definitely represents the relations between them, as so many objects of the Icon. Now necessary reasoning makes its conclusion *evident*. What is this “Evidence”? It consists in the fact that the truth of the conclusion is *perceived*, in all its generality, and in the generality of the how and the why of the truth is perceived. What sort of a Sign can communicate this Evidence? No index, surely, can it be; since it is by brute force that the Index thrusts its Object into the Field of Interpretation, the consciousness, as if disdaining gentle “evidence”. No Symbol can do more than apply a “rule of thumb” resting as it does entirely on Habit (including under this term natural

disposition); and a Habit is no evidence. I suppose it would be the general opinion of logicians, as it certainly was long mine, that the Syllogism is a Symbol, because of its Generality. But there is an inaccurate analysis and confusion of thought at the bottom of that view; for so understood it would fail to furnish Evidence. It is true that ordinary Icons, - the only class of Signs that remains for necessary inference, - merely suggest the possibility of that which they represent, being percepts *minus* the insistency and percussivity of percepts. In themselves, they are mere Semes, predicating of nothing, not even so much as interrogatively. It is, therefore, a very extraordinary feature of Diagrams that they *show,* - as literally *show* as a Percept shows the Perceptual Judgment to be true, - that a consequence does follow, and more marvellous yet, that it *would* follow under all varieties of circumstances accompanying the premisses. It is not, however, the statical Diagram-icon that directly shows this; but the Diagram-icon having been constructed with an Intention, involving a Symbol of which it is the Interpretant (as Euclid, for example, first announces in general terms the proposition he intends to prove, and then proceeds to draw a diagram, usually a figure, to exhibit the antecedent condition thereof) which Intention, like every other, is General as to its Object, in the light of this Intention determines an Initial Symbolic Interpretant. Meantime, the Diagram remains in the field of perception and imagination; and so the Iconic Diagram and its Initial Symbolic Interpretant taken together constitute what we shall not too much wrench Kant's term in calling a *Schema*, which is on the one side an object capable of being observed while on the other side it is General. (Of course, I always use 'general' in the usual sense of general as to its object. If I wish to say that a sign is general as to its matter, I call it a Type, or Typical.) Now, let us see how the Diagram entrains its consequence. The Diagram sufficiently partakes of the percussivity of a Percept to determine, as its Dynamic, or Middle, Interpretant, a state [of] activity in the Interpreter, mingled with curiosity. As usual, this mixture leads to Experimentation. It is the normal Logical effect; that is to say, it not only happens in the cortex of the human brain, but must plainly happen in every Quasi-Mind in which Signs of all kinds have a vitality of their own. Now, sometimes in one way, sometimes in another, we need not pause to enumerate the ways, certain modes of transformation of Diagrams of the system of diagrammatizaton used have become recognized as permissible. Very likely the recognition

> descends from some former Induction, remarkably strong owing to the cheapness of mere mental experimentation. some circumstance connected with the purpose which first prompted the construction of the diagram contributes to the determination of the permissible transformation that actually gets performed. The Schema *sees*, as we may say, that the transformate Diagram is substantially contained in the transformand Diagram, and in the significant features to it, regardless of the accidents, - as, for example, the Existential Graph that remains after a deletion from the Phemic Sheet is contained in the Graph originally there, and would do so whatever colored ink were employed. The transformate Diagram is the Eventual, or Rational, Interpretant of the transformand Diagram, at the same time being a new Diagram of which the Initial Interpretant, or signification, is the Symbolic statement, or statement in general terms, of the Conclusion. By this labyrinthine path, and no other, is it possible to attain to Evidence; and Evidence belongs to every Necessary Conclusion. (NEM IV, 316-19)

The remainder of this chapter tracks the implications of this passage, partly in terms of its relation to Peirce's thought, partly in terms of the actuality of its contents.

The Diagram as an Icon of Rationally Related Objects

The diagram is a skeleton-like sketch of its object in terms of relations between its parts, but what makes it apt to reason with, to experiment on, respectively, is the fact that it is constructed from rational relations. In this requirement, Peirce explicitly continues a Kantian requirement of the foundations of science: the schematism. In Kant, the finitude of man entails that we have no access to 'intellectual intuition'; we can not - as may the gods - intuit the object in itself; we may only approach the object in a pincer movement with two flanks: concepts and intuitions, respectively. Concepts without intuitions are empty; intuitions without concepts are blind, as the well-known Kantian doctrine goes. The two may meet only in schemata, a priori as well as a posteriori, and the former constitute the condition of possibility for the famous synthetic a priori judgments.[xi] Kant's central examples are mathematical: arithmetic is the schema rendering the concept of quantity intuitive, while the schema of the triangle is what permits an unlimited series of empirical triangles to be subsumed under the triangle concept. Peirce's demand that the relations in the diagram be rational is inherited from Kant's synthetic a priori judgment notion, just like his idea that rationality is tied to a generalized subject notion: rational relations are those known by "anybody who reasons". As is evident, Kant's "transcendental subject" is pragmatized in this notion in Peirce, transcending any delimitation of reason to the human mind: the "anybody" is operational and refers to anything which is able to undertake reasoning's formal procedures. In the same way, Kant's synthetic apriori notion is pragmatized in Peirce's account:

> Kant declares that the question of his great work is "How are synthetical judgments a priori possible?" By a priori he means universal; by synthetical, experiential (i.e., relating to experience, not necessarily derived wholly from experience). The true question for him should have been, "How are universal propositions relating to experience to be justified?" But let me not be understood to speak with anything less than profound and almost unparalleled admiration for that wonderful achievement, that indispensible stepping-stone of philosophy.
> ("The Logic of Quantity", ch. 17 of "Grand Logic", 1893, 4.92)

Synthetic a priori is interpreted as experiental and universal, or, to put it another way, observational and general - thus Peirce's rationalism in demanding rational relations of the diagram is connected to his scholastic realism posing the existence of real universals. The relations which make up the diagram are observational and universal at one and the same time, and they constitute the condition of possibility for the diagram to exist as an icon (observationality) with respect to which it is possible to entertain generally valid experiments (universality). The extension of this concept of rational relations is only described negatively in Peirce's account; in a parallel version to the "PAP" quotation above, he says:

> But we do not make a diagram simply to represent the relation of killer to killed, though it would not be impossible to represent this relation in a Graph-Instance; and the reason why we do not is that there is little or nothing in that relation that is rationally comprehensible. It is known as a fact, and that is all. I believe I may venture to affirm that an intelligible relation, that is, a relation of thought, is created only by the act of representing it. I do not mean to say that if we should some day find out the metaphysical nature of the relation of killing, that intelligible relation would thereby be created.

> [...] No, for the intelligible relation has been signified, though not read by man, since the first killing was done, if not long before. (NEM IV, 316n)

Peirce's pragmatizing Kant enables him to escape the threatening subjectivism: rational relations are inherent in the universe and are not our inventions, but we must know (some of) them in order to think.[xii] The relation of killer to killed, is not, however, given our present knowledge, one of those rational relations, even if we might later become able to produce a rational diagram of aspects of it. Yet, such a relation is, as Peirce says, a mere fact. On the other hand, rational relations are - even if inherent in the universe - not only facts. Their extension is rather that of mathematics as such, which can be seen from the fact that the rational relations are what makes necessary reasoning in diagrams possible - at the same time as Peirce subscribes to his father's mathematics definition: Mathematics is the science that draws necessary conclusions - with Peirce's addendum that these conclusions are always hypothetical. This conforms to Kant's idea that the result of synthetic a priori judgments comprised mathematics as well as the sciences built on applied mathematics.[xiii] Thus, in constructing diagrams, we have all the possible relations in mathematics (which is inexhaustible, following Gödel's 1931 incompleteness theorem) at our disposal. Moreover, the idea that we might later learn about the rational relations involved in killing entails a historical, fallibilist rendering of the a priori notion. Unlike the case in Kant, the a priori is thus removed from a privileged connection to the knowing subject and its transcendental faculties. Thus, Peirce rather anticipates a fallibilist notion of the a priori (cf. ch. 8). In the alternative PAP version, Peirce continues: "At any rate, a Diagram is clearly in every case a sign of an ordered Collection or Plural, - or, more accurately, of the ordered Plurality or Multitude, or of an Order in Plurality" (ibid.). We can say that the diagram is so to speak the redrawing of an icon in terms of a priori relations between its parts. In contrast to the wider term icon, defined by its relation to the object, the subcategory diagram is thus defined through its mode of rationally representing:

> The Diagram represents a definite form of Relation. This Relation is usually one which actually exists, as in a map, or is intended to exist, as in a Plan. But this is so far from being essential to the Diagram as such, that if details are added to represent existential or experiential peculiarities, such additions are distinctly of an undiagrammatic

> nature. The pure Diagram is designed to represent and to render intelligible, the Form of Relation merely." (ibid.)[xiv]

Thus, it is possible in a diagram to dissociate the pure diagram, built from rational relations, on the one hand, and its application: what the diagram may, in turn, be used to signify (via symbols) or refer to (via indices) on the other. Thus, the pure relational diagram forms a type[xv].

The Diagram as Type

Taken separately from its signification and reference, a diagram is itself a type. Consisting of rational relations, it is no wonder that the diagram as such is an ideal entity which is, in turn, communicated through particular diagram tokens. The diagram in itself is not the graphic figures on the sheet before us or before our innter gaze, as we might spontaneously believe. The diagram-icon should not be perceived as a particular figure: already before ascribing to the diagram any content or reference whatsoever, there is a crucial process of abstraction (in Peirces' terminology, prescission, see ch. 11) taking place, allowing the particular sinsign to be interpreted as instantiation of a type by bracketing all accidental features of the token at the profit of the type: "One contemplates the Diagram, and one at once prescinds from the accidental characters that have no significance" (NEM IV, 317). When seeing a geometrical figure drawn on a blackboard, we immediately prescind from the stripe of chalk having any breadth, from the line's vacillating deviation from linearity, from the particular color of the drawing, and so on. This type-reading of a diagram token now depends on the set of rules, explicit or implicit, that is selected to govern its typicality. Thus, one and the same diagram token may be read as a type in widely differing ways according to the rules of interpretation used. A line may be interpreted in one diagram as a borderline, in another as a line of connection between two points, in yet another as a transport of some object between two locations. This may be banal, but nevertheless it is an important feature in the diagram's iconicity: the type only becomes apparent in light of the use of certain rules - long before the virtual application of the diagram on more specific meanings, not to talk about empirical reference. This implies that already the pure diagram is an icon governed by a rule, that is, by a symbol. For instance, the sinsign ○ may be read as a token of the type *circle*, as a token of the type *circular disc* (including its interior), of the type *circular hole* (excluding its interior), of the type *conic section* (any other conic section, a point, an ellipse, a parabola, etc. would do as well as token), of the type *Jordan-curve* (a closed curve; here any other closed curve, e.g. a rectangle, would fulfill the purpose), of the type *hole in a two dimensional surface* (a hole of any other shape would do as well), of the type *topological sphere in 2 dimensions*, of the type *closed and connected manifold*, etc., etc., - each of these choices, in turn, yield different possibilities of which content the diagram type may be used to signify. In the language of Hjelmslevian semiotics, we could say that the diagram token is a unit of the expression substance referring to different types at the form-of-expression-level - all prerequisite to any reference to types in the content plane. Thus, the diagram type consists of two parts: a diagram token and a set of reading rules for the understanding of it as a type (which may, in many cases, be implicit); thus on the level of pure diagram types, the Kantian intuition-concept (talking Peircean: observation-universality) duality, is present in the very construction of the diagram as sign.

The Diagram as the Interpretant of a Symbol

In the next step, this diagram type only becomes a diagram *in actu* (recalling Peirce's basic dictum that signs are only signs *in actu*) when it becomes part of the inference process. To this end, the diagram type needs to be endowed with a symbolic signification - it must involve a "Symbol of which it is an Interpretant". Of course, it was only possible to construct the diagram type in the first place by precisely such a symbol (the reading rules just referred to), but these were all on the purely rational, pre-empirical level. The diagram being constructed as a type due to this symbol (the circle above e.g. taken to mean *totality* in a Neurathian cake-diagram), may now, in turn, act as the interpretant of another symbol (the population of Denmark, e.g.). The symbol in question refers to a general object while the diagram in question - being an iconic legisign, a type, - is in itself one. The condition of possibility for this connection is thus the generality of both terms; the diagram being a type and the symbol referring to it being general as to its object. This connection forms the defining semiotic link of the diagram. As the symbol refers to a general object while the iconic legisign, the diagram type, is in itself one, the possibility of the diagram lies in letting the latter constitute the signification of the former and hence refer to the same object. Of course, this is no merely arbitrary connection; what Peirce does not explicitly emphasize in this context (but elsewhere) is the fact that any symbol which is not a completely empty convention must always already refer to some icon (or, at least, it must make possible a process of inference leading to an icon), this icon being its initial interpretant before the symbol might be further elaborated in a diagram. The construction of the diagram, then, amounts to substituting for the initial interpretant of the symbol - the *Vorverständnis* of it, so to speak - a more precise and relationally elaborate icon.

This is a crucial point in order to understand the diagram's double determination - iconic and symbolic, perceptive and general - in Peirce. The diagram is an icon, but a special icon insofar as it is governed by a symbol, and in many cases doubly so, governed both by the type of rational relations used and the empirical phenomenon referred to (like the circle and the Danish population). But what does it imply to be governed by a symbol, to be the signification of a symbol? A symbol is defined by denoting a kind of thing, that is, an idea, not a particular thing ("Art of Reasoning", 1894, EPII, 9; 2.300); it does so by connecting a set of (possible) indices to an icon ("The Short Logic", 1895, EPII, 17; 2.295); it is a law, or regularity of the indefinite future ("Syllabus", 1903, EPII, 274; 2.293), and this implies that it is a rule which will determine its interpretant (ibid., 2.292). It is, simply, a sign making explicit its interpretant, its signification (this in contradistinction to pure icons and indices, respectively). It is a sign referring to all possible entities acting according to some rule which is described by means of an icon: "It is *applicable to whatever may be found to realize the idea connected with the word* ..." ("The Art of Reasoning", 1894, EPII, 9; 2.298), and the habit or rule defining it links together icons: "A *Symbol* is a sign which refers to the Object that it denotes by virtue of a law, usually an association of general ideas, which operates to cause the Symbol to be interpreted as referring to that Object." ("Syllabus", 1903, EPII, 292; 2.249). But the symbol does not determine the particulars which fall under it - except from precisely their falling under it. This is why it is necessarily general, and thus vague as to its extension. But it may also be vague as to its intension because being defined by a rule connecting icons: these need not be clearly defined, as is most often not the case in non-scientific concepts. Thus, the concept "dog" is vague because it is not possible to determine beforehand all single creatures it may apply to now and in the indefinite future, but it is also vague for the reason that there is no sharp borderline between it and the concept of wolf. But still, it is defined by a rule-bound association of icons, constituting a general kind. Now, as is evident from these deliberations, any symbol in itself always already constitutes a proto-diagram, insofar as its predicative aspect is iconic. Peirce emphasizes this in "New Elements"(1904): "A *diagram* is an *icon* or schematic image embodying the meaning of a general predicate; and from the observation of this *icon* we are supposed to construct a new generel predicate." (NEM IV, p. 238). The rule in it needs not be explicit, as it is appropriately hinted at in the identification of rule with habit in Peirce. The diagram, then, can be seen as the making explicit (some of) the habits already inherent in a symbol.

Of course, it is important to keep in mind that the mode of existence of the symbol's object is here bracketed; it may refer to existent, future, past, imaginary, fantasy, or any other objects. The symbol "unicorn" is no less a symbol because its object does not exist. It is perfectly possible to let a diagram make explicit the content of a symbol whose referent is fictitious merely. On the other hand, it is an important diagram property that it is beyond the reach of any diagrammatization to picture inconsistent symbols; this constitutes the very strength of diagrammatic formalization: every (correct, that is) diagram corresponds to a possibility[xvi]. For instance the grammatically correct symbol "round square" which implies a rule connecting the two iconic qualities "round" and "square" reveals itself as inconsistent precisely when we try to construct a diagram to express these properties in one and the same figure[xvii]. The same goes for more complicated and less intuitive cases, for instance "the rational square root of 2"; here a more complicated diagram is needed in order to grasp that symbol's inconsistency. Briefly, being an icon, the diagram can not be inconsistent. It may display non-existent entities, but not logically inconsistent entities. Its object is necessarily *possible*[xviii]- in contrast to the object of a merely symbolic expression. This constitutes a basic motive for diagrammatic reasoning: it can make explicit (parts of) the signification of a symbol and pragmatically weed out symbol inconsistencies.

Similarly, no distinction between more or less empirical symbols rules out explication by means of diagrams: both may give rise to diagrammatic explication. There are, of course, prototypical cases of pure and empirical diagrams, respectively, cf. for instance a diagram representing various parts of a population in a cake-diagram vs. a drawing of a circle as a diagram for the concept circle.[xix] A pure diagram will be purely mathematical (for instance a map with no reference to its empirical interpretation but only referring to a 2-D surface with certain structures on it), while an empirical diagram will be the interpretant of some empirical symbol in the actual or some possible world (for instance, a topographical map of a country, fictive or not). This must not be confused[xx] with the question of reference which is the issue whether the diagram is used in a proposition (a Peircean "dicisign"), that is, applied to objects referred to by indices (for instance, a map of England). Thus, the empirical case covers two subcases: one where the diagram depicts relations of a material ontology with no factual content, the other when empirical facts are also represented in the diagram by means of indices.

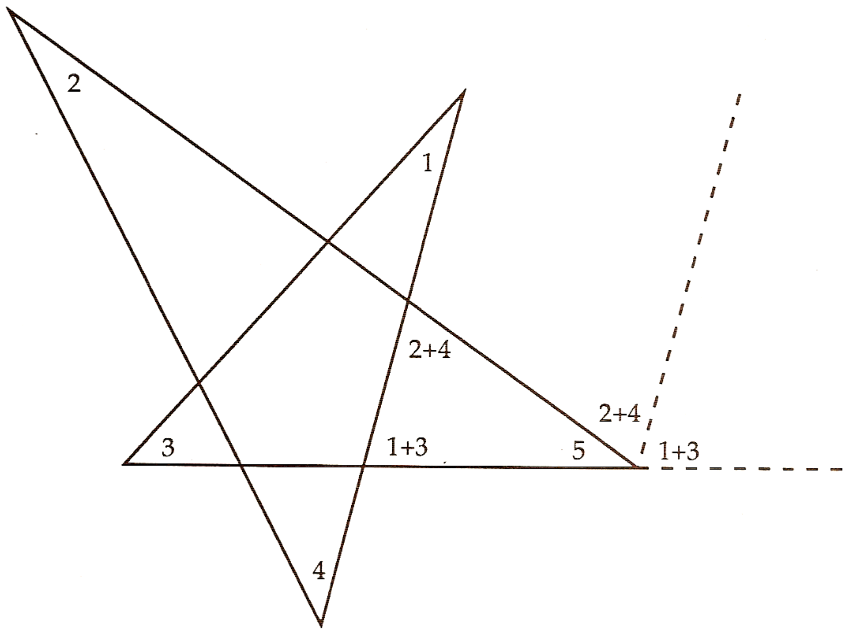
Thus, the diagram may make explicit the consistent content of (parts of) both more and less general symbols - and these may in turn be used as predicates in propositions about indexically identifiable subjects (which also may be general, to be sure).

The Diagram as a Formal Machine for Gedankenexperimente

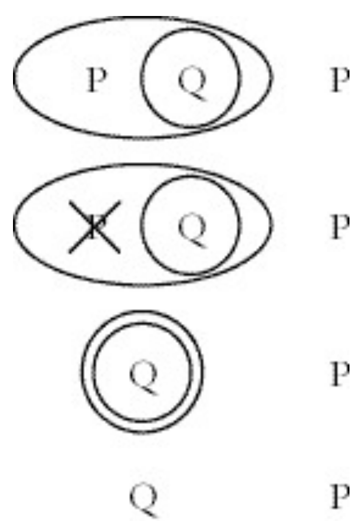
Now, we reach the core point of Peircean diagrammatology: the diagram as vehicle for mental experiment and manipulation. The operational definition of the icon is intimately tied up with diagrammatic experimentation. Let us take a closer look at these connections. The central phase of the diagrammatic reasoning process, motivating the very construction of the diagram, is *deduction*: the demonstration of the fact that a certain version of the diagram necessarily follows from another. Thus, every deduction is diagrammatic of nature and the logic of diagrams is an extension of the traditional concept of deduction (tied to truth-preserving operations in symbolic logic) to cover a large range of phenomena not usually considered as deduction (that is, unless translated into the symbolic form of a formal language) - but describable as such in so far as they qualify as necessary movements of diagrammatic thought.

A constructive geometrical proof is probably the arch example of a diagrammatic experiment – we have already in the Introduction discussed a simple and often quoted example - the diagrammatic version of Pythagoras.[xxi] The proofs given in classical Euclidean geometry may serve as core examples as they indeed were for Peirce who took them to be prototypical cases of diagrammatical reasoning.

Let us consider a couple of other examples. Diagrams play a huge role in both the research and the teachings of mathematics[xxii]. It is interesting that many proofs can be given in almost pure diagram form (with little or no accompanying text), such as the following proof of the angle sum of a star's vertices being 180 degrees:



Here, the angles are indicated by numbers. By seeing that angles facing the opposite way when a line crosses two parallel lines are equal, the sum of the angles in the small triangles to the right (which we know to be 180 degrees) can be seen to be equal to the sum of the star point angles. This, of course, involves the imaginary and experimental transformation and adding of angles – here strongly guided by the labeling of angles.



Another example can be taken from Peirce's own Alpha graphs – the proof of modus ponens in that system. The left side of the graph is a "scroll" representing the implication P -> Q – while the right side represents "P is true". The simple rules of Peirce's system now permits us to experiment with the diagram. The appearance of a graph in an outerly area permits the erasing of it from a more innerly area, so we are permitted to strike out the P in the implication, leaving us with Q in a double cut. But another rule – basically corresponding to double negation equaling assertion – permits us to erase the double cut, leaving Q alone on the basic sheet which equals "Q is true".

Perfectly simple– and somehow deludingly so - examples both of them, not implying the ingenious invention of new diagram manipulations or the introduction of new auxiliary objects into the diagram. The former is continuous, requiring the imaginary translation of geometrical objects on the surface; the latter discontinuous, involving the adding or erasing of whole discrete structures on the sheet. Thus, diagrams comprise both continuous and discontinuous variants – the latter traditionally experienced as more logical – but without any assumption of the latter serving as deep structure of the former. In both cases, diagrams facilitate, that you, thanks to the "typical" idealization in the diagram, are able to work directly on the ideal state-of-affairs in question.[xxiii]

Another prototypical example is the solution of an equation during a series of well-controlled steps according to the transformation syntax given by elementary arithmetic – mirrored by the solution of the same equation given its graphical representation thanks to analytical geometry.

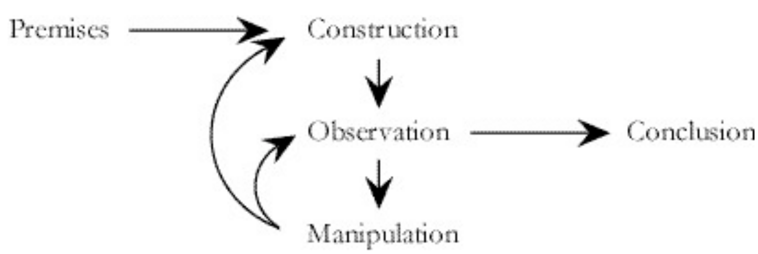
These are experiments on pure diagrams - prior to indexical and (empirical) symbolical reference, but once an empirical diagram is constructed, the experiment follows the same formal procedure. A map permits you to find a route between two given localities (there is no unique solution, but any line connecting the two is necessarily one). An economic growth graph in a business magazine permits you to determine the actual tendency. These experiments are very simple, indeed, but the important thing is there is a continuum between such examples on the one hand and very difficult, even yet unsolved, problems in mathematics, on the other.[xxiv] A very crucial observation here is that empirical diagrams continuously shade into ordinary icons. Take a photograph of a tree - it is an icon in so far as not previously explicit information may be gathered from it - say, e.g., the fact that the crown of the tree amounts to two thirds of its overall height. This fact was remarked nowhere earlier, neither by the photographer nor the camera nor the developer - and by noticing it you performed a small experiment of diagrammatic nature: you took the trunk of the tree and moved upward for your inner gaze in order to see it cover the height of the crown twice, doing a bit of spontaneous metric geometry, complete with the implicit use of axioms like the invariance of translation. Of course, this is an ordinary icon in so far as nobody constructed it with a diagrammatic intention. Nevertheless, you used it - *in actu* - that way. This continuum between diagrams proper (be it pure or empirical) and diagrammatic use of ordinary icons shows the centrality of the diagram for the icon category as such. It is with diagrammatic means that the operational use of the icon proceeds. Still, a distinction must be maintained between diagrams proper - that is, diagrams constructed with the explicit intention of experimentation and endowed with an explicit or precise syntax of transformation - on the one hand, and on the other, the more comprehensive class of diagrammatic unfolding of information from more "innocent" icons. In any case, this defining feature of the diagram - its possibility of being rule-bound transformed in order to reveal new information - is what makes it the base of Gedankenexperimente, ranging from routine everyday what-if to scientific invention.

A famous example is the German chemist August Kekulé's discovery of the stereochemical arrangement of the Carbon atoms in Benzene (C_6H_6), forming a ring. The composition formula was well known, but it remained an enigma how six Carbon atoms, Carbon having a valence of four, might form a molecule with only six Hydrogen atoms. A normal Carbon chain would leave too many unsatiated connections in order for only six Hydrogen atoms to complete the molecule. According to his description in his 1890 25 years celebration speech, the scientist sat in 1863 daydreaming before the fire, exhausted by speculation. He then saw one of the flames assuming the figure of a snake which turned around and bit itself in the tail to form a ring-like structure which wiggled contemptuously before his gaze - and all of a sudden, Kekulé realized that the normally linear carbon chain in the Benzene case turned around to form a circle.[xxv] That discovery thus formed a spontaneous case of diagrammatical reasoning, realized in the shape of metaphors. The flame was taken as ametaphor of the snake which, in turn, was taken as a metaphor of the carbon chain – a structure of metaphors held together by the common diagram of a piece of line, able to bend. The spontaneous diagram experiment argued that the Carbon chain, just like a snake, was able to form a ring, and subsequent chemical analysis corroborated the idea, leading to a major breakthrough in organic chemistry.

The Diagrammatic Reasoning Process

Before the crucial diagrammatic transformation is undertaken, however, preliminary steps take place in the overall picture of diagrammatic reasoning. The initial diagrammatic intention is in itself an interpretant of a Symbol (Peirce here refers to the Euclidean procedure of beginning with a statement of the general proposition to be proved, and then drawing a figure to illustrate the premiss of the conclusion). Thus, the reasoning process begins with the drawing of a diagram to exhibit the antedecent condition of its object, determining an "Initial Symbolic Interpretant". These two, taken together, now form the Peircean equivalent to the Kantian schema: the drawing constitutes its observable side; the initial interpretant its universal signification. Take an example: the drawing of a bridge construction equipped with the appropriate equations pertaining to its carrying ability. After this initial phase, Peirce in the long quotation above (PAP) considers the middle phase, albeit in a strange psychological tone alien to him: this initial schema determines "a state of activity in the Interpreter, mingled with curiosity. As usual, this mixture leads to Experimentation." Yet, he immediately admits that such a development must take place in any semiotic Quasi-Mind; we may discern the phenomenological core in the psychological shell: the central feature is the equipment of the initial diagram with transformation possibilities. Peirce here considers the sources for the transformation syntax: "... certain modes of transformation of Diagrams of the system of diagrammatization used have become recognized as permissible. Very likely the recognition descends from some former Induction, remarkably strong owing to the cheapness of mere mental experimentation." One source for transformation rules thus comes from the diagram itself, and their deductive status untold, Peirce refers their recognition to "some former induction" (say, the law of gravity involved in the equation system for the bridge's carrying ability has been corrobated by induction). This "former induction" must, in fact, be taken to refer to at least two separate sources. First, what we introduced above as the symbol's pre-diagrammatic immediate interpretant; including the idea inherent in the symbol of certain developments being possible for its object, others not so. This signification is also iconic, even if not explicitly diagrammatic; it constitutes so to speak our common-sense *Vorurteil* as to the content of the Symbol which the diagram more rationally illustrates, in our example, the common-sense understanding of a bridge (implying that we gauge the effect of normal, mesoscopic vehicles, not planets or atoms, on it). But the vagueness here probably comes from the fact that the determination of the possibilities of experimentation on the diagram is twofold and has yet another source of transformation possibilities in addition to the vagueness inherent in the symbol's generality. Quite another comes, namely, from the very structure of the diagrammatic figure as a legisign (without any reference to which possible symbols it may be taken as an interpretant of): which purely formal possibilities does the law governing the sign allow for varying upon the diagram's arrows, amount of entities, forms, structure, etc.; in the bridge case the set of equations with variables taken separately, apart from their actual referent. The former has its source in the generality of the symbol's object, the latter in the generality of the diagrammatic sign itself - what Peirce calls its being a type. Peirce identifies one more source stemming from the diagram intention (the fact that we want to gauge our bridge's carrying ability) which makes us experiment in order to fulfill this intention (we may vary the weight carried in order to find the point where the bridge can carry no more, hopefully far above the average weight of expected vehicles).

After having performed the transformation, in any case, the transformate diagram now displays the result at the same time as it is evident that the transformate diagram was contained in the transformand diagram. The transformate diagram, the eventual, rational interpretant of the transformand diagram, has in itself the conclusion, expressed in symbolic terms, as its interpretant: the bridge may carry vehicles up to 100 tons. Thus, the steps in diagrammatic reasoning lead from an initial symbol through three consecutive phases of diagrams and to a final symbol. We may envisage the possibility that the diagram transformation chosen does not lead to the expected result so that a trial-and-error process undertakes a new experiment on the same diagram. Say, if the bridge is shown to carry only 100 g vehicles, a new experiment changing the size or the material (or the diagram construction) may give a better result. Michael May has presented this model for the core processes of diagrammatical reasoning (May 1999, 186):



The diagrammatical reasoning process

Here, the shortcircuit leading from premiss over construction and observation to conclusion corresponds to corollarial reasoning, while the longer process involving one or several phases of manipulation and maybe even further construction corresponds to theorematical reasoning.

But there is a lot of possible prerequisites to be added to this ideal-type diagram transformation. First, the initial symbol already has its interpretant partly consisting of iconic material. (In our example: data about the bridge). Thus, the diagram may be said to be a rational analysis of parts of this pre-diagrammatic icon. But this entails the possibility of fallacies, if a diagram not conforming to the initial interpretant is chosen. Thus, experiments already in this phase may in some cases be expected; in cases less simple or less well-known than bridge building we might resort to a trial-and-error process here, experimenting with different formalisms in order to find those fitting best the intention. In that case, the deductive diagram transformation "molecule" just described becomes a phase in Peirce's overall heuristics. An initial abduction makes a guess about how to formalize a given phenomenon, the deductive diagrammatic phase just described follows, and finally an inductive investigation concludes the picture, in which the diagrammatic result is compared to the actual empirical data: Does the diagram transformation actually, in some sense, correspond to an evolution in the phenomenon mapped in the diagram?

We can sum up the steps of the process as follows:

> ***a*** *Symbol (1)*
>
> ***b*** *... having a rule-bound, initial, pre-diagrammatic, immediate iconic interpretant*
>
> ***c*** *Initial interpretant transformand diagram-icon* *(**a** + **c**) constituting the initial diagram, the "Schema")*
>
> ***d*** *Middle interpretant: the symbol-governed diagram-icon equipped with possibilities of transformation (with two sources, **a** as well as **c**)*
>
> ***e*** *Transformate diagram. Eventual, rational interpretant*
>
> ***f*** *Symbol (2) (Conclusion)*
>
> ***g*** *... having a post-diagrammatical interpretant differing from **b**. This interpretant being an interpretant of a as well, the diagrammatic reasoning has now enriched the total interpretant of the concept **a**.*

Thus, the process begins with "some former induction" having given rise to the initial symbol's pre-diagrammatic interpretant, an inductive generalization sedimented as the meaning of the symbol. This meaning, of course, must be to some extent already structured,[xxvi] and some of its rationally formalizable relations are now abductively selected, yielding a guess of which invariant properties may be sufficient to account for other central properties in the general object in question. Then, in the very construction process of the initial diagram, a constant feed-back comparison must take place between the general object as it is preliminarily and inductively grasped on the one hand, and on the other, the abductive guesses trying to establish against this background a more formalized diagram; in many cases this may take place almost automatically, due to the existence of well-established diagrams. It goes without saying that the fertility of the specific diagram chosen can only be fully measured with regard also to the deductive experimentation taking place later in the diagrammatic reasoning process which consequently also has a role in this constant feed-back trial-and-error process. But the overall picture of the initial phase of diagram construction is thus: the general knowledge contained iconically in the symbol - no matter if it be an empirical symbol or a mathematical one - is interpreted in relational terms so as to give (part of) the consistent general meaning an iconical illustration able to be manipulated, an illustration which is, on its side, also general. After few or many repeated transformations (subject to the three different sources of transformation syntax mentioned above), a transformate diagram is reached - its finality is of course only measured on how it accords with the initial intention. The evaluation of an interpretant for a candidate for final diagram status is in itself an abduction proposing a symbolic reading of that diagram, and this may, in turn, be inductively compared with empirical information present in the initial interpretant.

To sum up, the overall picture of the diagrammatic reasoning process is that it forms a formal deductive reasoning core, embedded, on each side, in the trial-and-error of abductive trials and inductive tests.

Cartography as an Example

Maps are no doubt a good candidate to a diagram subcategory: rule-bound depictions of aspects of the shape of phenomena. The non-trivial icon definition is evident here: the construction of a map, be it based on triangulation from a set of selected measuring points in the landscape, or on the rational, stylized rendering of aerial photography, does not explicity contain all information held in the map. What types of experiment may be performed with respect to a map that reveal this information? We may, for instance

1) find a route between two localities, 2) determine a distance or an area, 3) recognize landscape forms - and so on.

Of course, there is nothing very “experimental” in a laboratory sense of the word in these transformations; nevertheless their status as diagram transformations are granted by their fulfilling the demand for revealing truths not stated in the construction of the diagram. Take for instance the distance between two cities. To measure the map distance with a ruler and figure out the approximate real distance from the map's scale is a typical map manipulation, depending on the fact that the map we imagine here is endowed with a metric topology. In this case, a middle interpretant will be the map with your route on it added; the transformate diagram will be the map with the ruler - and the final conclusion will be of the form "The distance between New York and Pittsburgh is so-and-so-many miles", revealing a number nowhere present in cartographic triangulation nor aereal photography. The experiments possible of course depend on the type of map projection, some are area preserving but not distance preserving, some vice versa, some are distance preserving in some directions, not in other. Thus, different map types may be described simply with reference to which types of experiments they allow. Other maps do not even have a metric topology[xxvii], take for e.g. your typical subway map which does not keep invariant neither distance nor geometric form but which merely keeps invariant certain connexity properties: the connexity of the single subway lines - very often symbolized with one color for each - and the crossings and touchings of several subway lines, highlighted by a circle or other closed curve with another colour indicating the weaker connection possibilities of changing line. Here, it is easy to find one's localization and path relative to fixed points: the stations - but it is no longer possible to gauge metric properties (how far are we from the main station?) nor morphological properties (a curved track may be represented by a straight line, and vice versa, even in the same map), nor sub-area categorization. In this case, the initial schema will be the colored spaghetti articulation interpreted as a diagram-icon by the symbol "London Underground", the middle interpretant will be your present position and the end of your travel, and the transformate diagram will be the possible paths between the two tracked as continuous lines on the spaghetti figure, and, again, the conclusion will be symbolic statements of the type "We gotta change at Piccadilly"; "There's no way there without changing two times", "It seems to be the shortest way to go via Victoria" (shortest here referring to transport time measured by means of number of stations passed, rather than to any metric property of the diagram).

Error! Objects cannot be created from editing field codes.

Even in as simple diagrams as road maps, we can appreciate the distinction between two experiment classes. One is the simple use of the diagram, following the transformation rules more or less explicitly given. Another is experimental in a stronger sense of the word: it experiments with the very layout of the diagram itself: the possibility of building a new subway line in order to mend London's traffic problems. These more ambitious experiments, in turn, may involve two dimensions: one is further information with respect to the object (or in our ideas about it) making the extension of diagram possibilities desirable. Another is the change in the very formal apparatus of the diagram (these two may, of course, trigger each other), as for instance the development of Venn diagrams out of Euler diagrams by the addition of a rule (the shading of an area referring to an empty set), or the reinterpretation of Euclid's axioms in order to make non-Euclidean geometries. Already the first type of experiment is unlimited as soon as the diagram in question is continuous (like most maps), but in sufficiently complicated diagrams we must expect not to be able to account for the possibilities of interesting experiments beforehand (cf. the Gödel inexhaustibility of mathematics). On two different levels, these properties are what constitute the well-known "depth" of icons and diagrams. This inexhaustibility is dryly remarked on by Peirce when he epigrammatically states about the content of a diagram simply that "Everything is involved which can be evolved." ("Logic of Quantity," from the "Grand Logic," 1893, 4.86)

The Generality of Diagrams

The strength of Peirce's diagrammatology lies in the extraordinary breadth covered by his diagram concept. In Jaakko Hintikka's Peirce interpretation, the audacity of Peirce's conception lies in his generalization of structures in geometrical reasoning to logic in general. To Hintikka, this generalization becomes evident in Peirce's distinction between "corollarial" and "theorematical" reasoning. Peirce himself called this "My first real discovery about mathematical procedure" ("Parts of Carnegie Application", 1902, NEM IV, p. 49) – named of course after corollaries and theorems in geometrical proofs. "Peirce's brilliant insight is that this geometrical distinction can be generalized to *all deductive reasoning*." (Hintikka 1983, p. 109). Corollarial reasoning refers to conclusions which may be read directly off the diagram, interpreted in the right generalized way – while theorematical reasoning, on the other hand, requires the introduction of auxiliary constructions not explicitly referred to in the premisses (in the initial diagram). In Euclid, such auxiliaries often have the character of the construction of further figures in the diagram – and in the geometrical tradition the distinction between corallarial conclusions and theorematic constructions is often spontaneously referred to as between logical and geometrical consequences (ibid.). In the standard conception of logic referring to formal languages, no such distinction is, of course, possible, all inferences being simply shorter or longer cases of symbol manipulation. In Hintikka's account, however, Peirce's distinction is not only due to superficial representation differences in geometry but refers to deep issues in logic. First, Hintikka emphazises that theorematical reasoning necessarily implies the introduction of new variables in the inference process – often in the shape of a lemma in mathematical proofs. This points to the non-triviality of theorematical reasoning as opposed to corollarial reasonings. But then theorematical reasoning becomes a matter of degree – depending on how many new individuals are introduced in the construction, and thus yielding a "rough measure of the nontriviality of an argument" (p. 113). Thus we may imagine a continuum between completely trivial corrolarial reasoning in one end and still more non-trivial theorematical reasoning in the other. Peirce's concept of diagram manipulation and diagram experiment thus refers to a whole range of theorematical reasonings of various degrees of non-triviality. Second, Hintikka emphasises that Peirce's distinction solves the age-old problem he nicknames "logical incontinence" (114): how can anybody possibly miss the knowledge of all logical consequences of one's premisses? Corollarial conclusions are indeed evident to everybody who is able, at all, to understand the diagram in question as a type. But theorematic conclusions require both the introduction of the right auxiliary entities and the right construction obtained from combining these entities with the intitial diagram. Theorematic reasoning thus may require huge skills on the part of the reasoner so there is no wonder why such conclusions may be difficult to obtain. Third, Hintikka argues that the distinction throws a new light on Kant's analytic-synthetic distinction. It is unclear whether Kant's concept of analytic judgment covers corollarial deductions only, or both corollarial and theorematical deductions, simply because Kant did not have a corresponding distinction at his disposal (115). Hintikka argues that Kant's concept of analytic inferences covers corollarial reasoning only, so that theorematical inferences must be classified as synthetic – corresponding to Peirce's idea of diagrammatical reasoning as covering the synthetic a priori domain. We shall return to this connection between diagrams and the synthetic a priori in chapter 8.

Hintikka's appreciation of the corollarial-theorematic distinction forms part of his overall charting of 20 Century logical thought in two main traditions, inspired by van Heijenoort's famous brief paper on logic as calculus versus logic as language. Hintikka's idea is that logic since the latter half of the 19 C has followed two different currents depending on basic ideas often not clear to the persons involved – currents involving huge parts of analytic and even continental philosophy. He nicknames them "language as Lingua Universalis" and "language as Calculus Ratiocinator" after Leibniz's famous dreams of a universal language and a reasoning machine, respectively.[xxviii] The main current in logic falls within the former, comprising Frege, Russell, Wittgenstein, the younger Carnap, Quine. Here, logic is a formalization of (parts of) language, and it is universal in so far as it is impossible to address the world by any other means than this one language. This has a series of corollaries. Logic refers to the one existing world only (hence the hesitation to accept counterfactuals); semantics becomes ineffable and impossible to formalize because it must be discussed in that same language which is the object of the discussion; formal syntax becomes the only objective of logic; linguistic relativism threatens because there is no possible corrective to the given linguistic understanding of something; the relation between world and language is impossible to attack outside the one universal language (Wittgenstein: the limits of my language and the limits of my world coincide), and hence no truth definition may be given. Surprisingly, what might at first glance seem to be a strongly realist position (logic as one, universal language) thus holds a series of antirealist or even relativist or skepticist consequences – becoming evident e.g. in Quine's linguistic holism and his ideas of the impossibility of translation. The alternative tradition has rather been an undercurrent running from Boole via Peirce, Schröder, Löwenheim to certain aspects of Hilbert, Gödel, the later Carnap – and, of course, to Hintikka himself. According to this tradition, logic is no universal language, but rather a calculus aimed at problem-solving in a given domain of discourse.[xxix] Thus, there is a possible plurality of logics, and this tradition has given rise to modal logic, epistemic logic, and model theory – very often, hence, Hintikka nicknames this tradition the model theory tradition because of its obsession with semantics and the issue of which formalisms fit best a given content. While this tradition might at a first glance seem antirealist with its emphasis on many competing representation systems, its corollaries are, quite on the contrary, realist. Semantics is possible, because one fragment of logic or language may be unproblematically put to use in the discussion of another such fragment; truth definitions may be given (even sometimes in the same language as it refers to – Hintikka's own "independence-friendly logics" with free combination of quantifiers as a logical example) and correspondence between representations and world is not only possible but may be continuously refined by the use of many different iconic representation systems for different domains of reality, depending on their pragmatical purpose. Thus, possible-world-semantics grows out of this tradition, just like the insistence of model theory that the semantics of a logical expression must be charted by means of the variation of its possible references. Both traditions embrace formalization, albeit for quite different reasons, which has added to the difficulty in distinguishing them. The universalist tradition embraces formalization because of the ineffability of semantics – which leaves formal syntax is the only or central goal to achieve. The calculus tradition, on the contrary, embraces formalization because of the clarity and efficiency of formal representation and of calculation – and formalization here may range from very general systems able to subsume highly different semantic interpretations (Hilbert who, maybe surprisingly, holds a stock in both traditions) and to highly semantically motivated formalizations in the other end, so to speak encompassing a continuum covering far more possible worlds than universalism's one-world claim in one end and covering highly specific discourse domains within that world in the other end. Hintikka's daunting hypothesis no doubt throws a new light upon the history not only of philosophy of the 20. century. The work of Martin Kusch (Kusch 1997) even points to the fact that also continental philosophy displays an exactly analogous split, Husserl falling on the language-as-calculus side while Heidegger falls on the universal-language side – thus placing Heidegger and Derrida along with Frege and Quine while Husserl sides with Peirce and Hintikka, quite different from standard history-of-ideas accounts. We shall return to this below. The anti-iconicity traditions discussed in ch. 3 thus appears in a new light given Hintikka's distinction – making it easier to understand how iconicity could possibly be attacked from two otherwise completely opposed camps of philosophy.[xxx]

Hintikka's account for Peirce's special role in the calculus tradition ("The Place of C.S. Peirce in the history of logical theory", in Hintikka 1997) resumes Peirce's obsession with interpreted logic, as opposed to axiomatized systems; his interest in modal logic; his anticipation of Hintikka's game-theoretical semantics in his ideas of the dialogicity of logical inference, his lack of hesitation towards formalizing logic by means of logic. Hintikka takes care to note that the plurality of possible representations does, in fact, point towards realism rather than the opposite:

> In general, like all believers in logic as calculus, Peirce was not only ready to provide an interpretation for their calculi at the drop of a symbol. He could – or thought they could – discuss such changes systematically in an explicit language. The very freedom of choosing a formalism differently on different occasions was sometimes a consequence of their belief that it is the underlying representational realities that really mattered, not the formalism. (Hintikka 1997, 150)

One could rather say that exactly the possibility of having competing representations of one and the same state-of-affairs is what grants realism – not unlike Marvin Minsky's old idea that intelligence requires at least two different representations of the same thing. This idea does not, as Hintikka takes care to note, in any way prevent Peirce, just like Hilbert, from being interested in a purely mathematical definition of his logic notation – the important thing is the possibility of explicitly discussing its interpretation. This points to the issue which is most important in our general diagrammatological context: the iconicity of logic. To Hintikka, Peirce's notion of iconicity and its idea of the sign as a model of its object is "not completely different" from the ideas of model theory. To Peirce – as opposed to the Fregean tradition's emphasis on thought at the expense of intuition – logic is, in itself, basically iconical. Hintikka here emphasizes Peirce's idea that "The Icon does not stand unequivocally for this or that existing thing, as the Index does ..." ("Prolegomena to an Apology of Pragmaticism", 1906, 4.531, quoted in Hintikka 1997, 153) – thus the icon necessitates interpretation and facilitates different uses (albeit, of course, not any possible interpretation or use). Hintikka sharply contrasts Peirce's iconicity of logic to the early Wittgenstein's picture theory of language: the latter is not only semantically ineffable and may only be showed, not explicitly discussed, but it is also a static mapping of propositions onto states-of-affairs. Peirce's iconicity of logic, on the contrary, does not only depict states-of-affairs but also their iconical interrelation, making reasoning using icons possible: it thus covers the very "iconic aspects of logical inference" (Hintikka 1983, 116) about which Wittgenstein necessarily remains silent. The very manipulations performed on a diagram do themselves iconically refer to possible experiments on the more or less ideal object referred to by that diagram – thus diagrams are not the accidental sums of iconical representations plus logical manipulation. They are icons depicting their object as well as the logical inferences performed on them.

Peirce's generalization of the geometrical concept of diagram to all of reasoning must be understood on this background. Diagrams are not Peirce's iconic equivalent to an all-embracing formal logic understood in the universal-language tradition. Diagrams is rather an umbrella notion for widely differing representation systems which may be picked, constructed and used for correspondingly differing domains and purposes – the diagram notion only insisting that all of them, despite all differences, possess an iconic basis and, in sufficiently complicated cases, makes possible diagram experiments in theorematic reasoning. Hence the concept of diagram in Hintikka's calculus interpretation is perfectly aimed at understanding processes of logical inference clothed in widely different representation systems and in widely different domains. In the latter half of this book, we shall investigate three such domains: biosemiotics, pictures, and literature.

Diagram Types

As in any branch of research, the possible establishment of an inventory of rational subtypes will constitute a major progress. Unfortunately, no simple diagram taxonomy seems to be at hand, at least not referring to pure diagrams - for the very simple reason that the category of pure diagrams is coextensive with mathematics as such. This implies that the question of pure diagram taxonomies is inevitably entangled in the large questions of the foundations of mathematics. Other taxonomies might refer to different diagram intentions, different diagram graphics, different diagram subjects, etc., but a comprehensive review of diagram taxonomies by Blackwell and Engelhardt (1998) reveals little agreement among scholars. Peirce, taxonomist of signs, never really attempts to develop a diagram taxonomy; the closest he gets might be the remark already quoted, made *en passant* in an early account for diagram experimentation in Robin (15), "On Quantity" (ca. 1895, in NEM IV, p. 275): "... a diagram, or visual image, whether composed of lines, like a geometrical figure, or an array of signs, like an algebraical formula, or of a mixed nature, like a graph ..." so that we might envisage yet another trichotomy comprising maps, algebra, and graphs; that is, simple diagrams, construction precepts, and diagrams equipped with construction precepts, respectively? The construction of a rational taxonomy of diagrams will be a major future challenge for (not only) Peircean semiotics[xxxi].

The Imaginary Moment in Diagrams: Peirce and Hilbert

During the operational interpretation of an icon, a certain phase typically appears which at the same time exposes the icon's full range of possibilities and displays a central danger of iconic fallacy. As discussed in ch. 2, this "imaginary moment" involves momentarily suspending the distinction icon-object while operating on the former, as Peirce notes in already in the 80's:

> Icons are so completely substituted for their objects as hardly to be distinguished from them. Such are the diagrams of geometry. A diagram, indeed, so far as it has a general signification, is not a pure icon; but in the middle part of our reasonings we forget that abstractness in great measure, and the diagram is for us the very thing. So in contemplating a painting, there is a moment when we lose the consciousness that it is not the thing, the distinction of the real and the copy disappears, and it is for the moment a pure dream -- not any particular existence, and yet not general. At that moment we are contemplating an *icon*.
> ("On theAlgebra of Logic", 1885, W5, 163; 3.362)

This moment of fiction when we, operating on the icon, takes it for the object itself, is crucial for our operations: here, the constraints on our operations stemming from the icon's formal properties are identified with the constraints stemming from the object's properties and the constraints stemming from the question leading us to diagram experimenting (the three sources discussed above), and it feels like we are operating on the very object itself. This goes for all icons, from paintings where we leave our observer's position and momentarily insert our imaginary body on a stroll into the landscape and to equations where we cease manipulating only ink symbols on a sheet and tackle invariances in arithmetic directly. This 'imaginary moment', of course, is a description in psychological terms of a phase in a process that is not itself of psychological nature. But the important thing in our context is the virtual source of error inherent in this moment: properties stemming from our preformed folk understanding of the object in question may interfere, without our attention, in our experiments with the icon - with the result that we see things in a picture not really presented there, or we find regularities in a formalism which are not really implied by it. The latter was, of course, the case in Euclidian geometry where our everyday conceptions prejudiced us to assume the parallel axiom true - a fact which in the history of mathematics predisposed mathematicians to be on guard against intuition.[xxxii]

Thus, there is a certain tension in this 'imaginary moment'. To the extent that the imaginary moment leads to the eventual interpretant, the conclusion seems to be directly "read off" the diagram and so furnishing evidence. On the other hand, even if this fertile moment is the very source of evidence, it is precisely the seductive welding together of object and representation in this phase which constitutes the major source of error in diagrammatic reasoning and has long since been recognized as such. The whole formalist endeavor in the philosophy of mathematics and the emphasis upon symbolic calculi and mistrust of geometry since the late 19. Century are based on attempts at getting rid of the dangers of seduction by intuition in this very moment. More precisely, this danger can be traced to the triple source of constraints on the possible experiments in this crucial phase of the reasoning process: they descend, as we saw above, from the initial pre-diagrammatic interpretant, from the diagram intention as well as from the internal regularities of the diagram-icon as iconic legisign. But the first two of these sources are of course ripe with common sense, with folk theories and virtually ideological preconceptions of the object and thus possibly with wishful thinking - and the imaginary moment may lure the reasoner into accepting these preconceptions and tacitly letting them govern the experiment so as not to discover crucial formal possibilities in the legisign or even to abandon internal legisign-constraints to the benefit of fallacious common sense assumptions in cases or aspects where the two are mutually exclusive. Hence the idea of formalism in mathematics; one could describe Hilbert's idea as that of getting rid of the imaginary moment precisely in the decisive part of the process leading from the diagram-icon via the middle to the eventual interpretant, bracketing the process from signification in these phases and then reinvesting it after having reached the transformed diagram, that is, the theorem. Of course, orthodox Hilbertians will be shocked to see the idea of purely formal proof theory (with intuition's role reduced to the level of meta-mathematical interpretation) transformed into iconic diagram manipulations: the Peircean process at first glance seems to be almost the opposite: one could leave out the possibly folk theoretic symbolic determination while manipulating the icon - and then reinvest the symbolic interpretation after having reached the theorem. But a closer analysis reveals the similarities: the diagram in Peirce is iconic indeed, but it is a formally controlled, "rational" icon equipped with a syntax of transformations, while the Peircean symbols here are the possible source of error because of their immediate interpretants in the form of pre-diagrammatic ordinary icons, "wild" icons, so to speak. The reason for confusion here comes from widely differing conceptions of "symbol".[xxxiii]

We can add that the well-known mainstream formalist idea from the full-fledged Hilbert doctrine of the 1920s that diagrams should be completely expelled from the proof to a role merely of heuristic support device (cf. Husserl below) was not always unanimous in Hilbert, as discussed in Michael Greaves' (2002) fine book on the somber destiny of diagrams in 20 C geometry and logic. Hilbert's famous standard idea as expressed in the early (1894) quote "A theorem is only proved when the proof is *completed independent of the diagram*." (72)[xxxiv] sometimes gave way to rather different ideas like the 1900 quote: "… arithmetical signs are written diagrams, and geometrical diagrams are drawn formulas" (74). Here, Hilbert in fact expresses a completely Peircean idea of equivalence between symbolic and diagrammatical expressions.

Correspondingly, formalist reading in Hilbert and rule-governed "imaginary moment" in Peirce may thus be seen as parallel ideas of controlling a seductive phase of reasoning. Then, the isolation of the purely diagrammatic part of the process (Peirce) would be equivalent to the idea of keeping a pure mathematical reasoning apart form uncontrollable iconicity (Hilbert).

Moreover, Hilbert perfectly realized that a certain and inevitable minimum of *Anschauung* remains indispensable even in symbolic calculation, namely the basic ability to identify, count, and permute symbols on a string. In both cases, then, the crucial opposition ceases to be between symbolic and iconic and becomes rather the opposition between a controllable, rational intuition and a 'wild' pre-formal intuition. The crucial difference is rather, now, that the Peircean point of view will see the remaining controlled domain of rational intuition as a definitively iconic field, while the Hilbertian will often see it as purely symbolic, unfortunately, but unavoidably, to be exposed to a severely constrained finite intuition, corresponding to the simplest arithmetic, able to infallibly count strokes in a row (assuming the early digital idea that the finite symbol alphabet could be translated into a system of such strokes). Of course, Hilbert himself was no Hilbertian and he perfectly realized the unavoidable remnant of *Anschauung* in this "formale Redeweise" (cf. Kreisel 1982).[xxxv] Here, Peirce's technical research into iconic logic diagrams shows, as mentioned, that the task undertaken by "symbolic" calculi may be equally well performed by apparently much more explicitly iconic systems.[xxxvi] The equivalence of Peirce's graphs to formal syntactical systems proves, of course, that the latter possess the same degree of iconicity as Peirce's – both may give rise to the extraction of the same amount of non-explicit information (as Hintikka also notes, Hintikka 1997, 154). Still, the problem that motivated Hilbert is still relevant for the Peircean account of diagrammatic reasoning: we cannot expect, even less can we demand, the imaginary moment to involve the whole process from initial interpretant to eventual interpretant. The very formal *raison d'être* of diagrammatic reasoning entails that purely diagrammatic constraints with no apparent interpretation, only motivated by the diagram as a rule-bound legisign, may take over in decisive phases of the argument and, just like in Hilbert, preclude 'wild' intuitions from intruding. Thus, the imaginary moment must be virtually split into two: an initial moment where diagram and symbol (1) are identified, and a final one where transformate diagram and symbol (2) are identified, so as to keep a pure diagrammatic transformation phase in between them. In this case, the comparison between symbol (1) and (2) of course becomes crucial: in the empirical case, the question will be, does the symbol (2) give meaning as expressed in a proposition about symbol (1) - e.g.: has an object of type (1) ever empirically given rise to an object of type (2)? If not then the diagram may be invalid, or the observation insufficient. So the pragmatist trial-and-error feedback between initial and final symbols in the diagrammatic reasoning process must be the Peircean means of avoiding being caught up in the 'imaginary moment'.

Diagram, Continuity, Concept, Abduction, Pragmatism ...

The diagram has intimate connections to many central aspects of Peirce's doctrine. The prototypical diagram: a set of lines between points on a continuous sheet of paper, may serve to indicate the important relation between the diagram as epistemological device and the signification of Peirce's notion of the continuum for metaphysics. How do we immediately "see" that the conclusion of a diagram experiment is valid for a whole class of cases referred to by the premisses? One source is, of course, the typicality of the diagram, but this typicality consists in the possibility of continuously deforming any token to the diagram type. Something analogous holds for the transformations. We see this by the fact that a continuum of possible realizations are built into the diagram. This may take place by different means: one is the continuity of the underlying sheet. By imaginatively performing the transformative change of angle size on the sheet we see that the tripartition of angles into acute, rectangular, obtuse is complete, because we can make the angle pass through all values between 0 and 180. The variable *x* is in the same way, so to speak, a hole in the sheet through which a whole continuity of instantiantions may pass. Of course, discrete diagrams exist where this idea is less relevant (equations defined only with reference to natural numbers etc.) - but continuity, so Peirce's metaphysics, is more inclusive than discontinuity, so that we are only able to understand the latter against the background the former.

Continuity is also, as we saw in ch. 2, the basis for Peirce's "medieval" realism with regard to the existence of real universals which refer to natural habits and the continuity of their possible instantiations. But diagrams are intimately connected to symbols, as we have seen, in the diagrammatic reasoning process. Concepts are "the living influence upon us of a diagram"[xxxvii] - this should be compared with Peirce's basic pragmatist meaning maxim, according to which the meaning of a concept is equal to its behavioral consequences in conceivable settings. This implies that signification of a symbol is defined conditionally: "Something is x, if that thing behaves in such and such a way under such and such conditions" - "Something is hard, if it is not scratched by a diamond". But this maxim, developed on the basis of a conception of scientific experimenting, is formally equal to the idea of diagrammatic experiments: the signification of the concept is the diagram of the experiment. The aim of science is to try to make such conditional definitions as diagrammatic as possible. This is the diagrammatic component in Peirce's laconic enlightenment maxim, "symbols grow": new symbols arise through diagrammatic experimentation.

Diagram Perspectives

Peirce readily admits that his use of the word diagram employs it in "... a wider sense than is usual" (PAP, 1906, NEM IV, 315); precisely this is the great advantage of his diagram concept: a whole series of semiotic processes - the tropisms studied by biosemiotics[xxxviii], the contemplation of pictures, metaphorical, analogical, and poetical reasoning, linguistic and narratological syntax, basic sensorymotor schemata, as well as mathematics proper - become understandable as different realisations of one and the same basic rational semiotic behavior, namely, diagram experimentation. Thus, it liberates semiotics from the static and narrow idea of the en- and decoding of signs, because the interesting part of semiotics lies elsewhere, in the epistemological dynamics of diagram interpretation - at the same time as it saves semiotics from the false and ungraspable "dynamics" of irrationalist poststructuralisms, vitalisms, and constructivisms. Moreover, it constitutes a wholly actual attempt at making explicit René Thom's great intuition in philosophy of science, " ... there is only science from the moment when you can embed the real within the virtual"[xxxix]. You only understand a phenomenon in terms of a scenario mapping (some of) the possible ways of *changing* that phenomenon into related, virtual phenomena. Quite contrary to Quine, eager to expulse counterfactuals from science, this basic idea is what diagrams formalize: various counterfactual transformations of the phenomenon's real possibilities as the means of gaining insight into it.[xl] The diagram as a central concept in epistemology thus unites a series of actual scientific and philosophical currents: cognitive semantics and linguistics, the resurfacing of diagrammatic reasoning in AI as well as, more generally, the renaissance of intuition, pragmatism, and scientific realism in the philosophy of science.

[i] See Roberts 1973, for a groundbreaking treatment of Peirce's logic graphs. Among recent diagrammatic scholars investigating Peirce's logic graphs could be mentioned John Sowa, Allwein/Barwise, Sun-Joo Shin and Ahti-Veikko Pietarinen. Shin claims the heuristic virtues of existential graphs and aims at rearticulating Peirce's basic graph operations for practical purposes, while Pietarinen argues for a philosophically deeper relevance of the "endoporeutic" (outside-in) interpretation direction of Peirce's graphs, the idea that the outermost layer of a diagram must be interpreted before going on to its interior. To Pietarinen, Shin's reformulation of Peirce's conventions attempting to render the graphs compositional goes against the outside-in reading of symbolic calculi and he points to Hintikkan game-theoretical, dialogical intepretations of logic in order to further develop Peirce's graphs. The outside-in reading, to Pietarinen, is important because it makes the interpretation depend on the context implicitly provided by the phemic sheet on which the graphs are drawn – the phemic sheet corresponding not to the universe as such, but to the universe of discouse, depending on tacit understandings between the dialogue partners (Pietarinen 2004, 128-30). Maybe Pietarinen's observation can be generalized to Peirce's diagram doctrine as such: the outside-in reading is preferred because taking the diagram as a gestalt informed by a context. In specific diagram interpretation, the outside-in reading will, of course, mix with inside-out readings in an ongoing trial-and-error process, but it is right for Pietarinen to insist that diagrams generally may not be presupposed to be compositional; compositionality rather forming a restricted subclass of special diagrams. Everyday examples of diagrams like a map of a country or a school timetable are evidently interpreted outside-in rather than inside-out.

[ii] Pure icons only exist as a limit category in Peirce – concrete signs being, as a rule, composite. "Hypoicon" is Peirce's notion, then, for signs whose mode of object reference is primarily iconic.

[iii] The force of this idea in metaphor analysis is obvious - and it is recognized, albeit in non-Peircean clothing, by the cognitive semantics tradition mentioned above.

[iv] In the development of Peirce's thought, the idea of a general diagrammatology thus precedes his construction of existential graphs rather then the opposite. It is his diagrammatology and his category phenomenology which permits him putting them to use in graphical logic representations as a special case.

[v] This fact is elaborated ingeniously in Hintikka 1997.

[vi] In addition to this basic, operational icon criterion, however, Peirce also has a continuous idea of measuring different degrees of iconicity of representations. Thus, he sees his own logic graphs as more iconic than symbolic representations because a variable is here presented by one continuous line of identity as against the repeated occurrence of a number of xs with the same reference in a symbolic representation. While the former preserves the unity of the variable, the latter represents it in a shattered way untrue to the unity of the reference of the variable. Thus, in his Beta graphs, he has two different ways of expressing identity between variables – one identifying them by menas of a continuous "identity-line", another identifying them by attaching the same letter ("selective") to them. The former Peirce sees as *more iconical* than the latter (even if the

latter may be heuristically superior). This points to another, *optimal* iconicity concept in Peirce in addition to the basic operational iconicity. See Stjernfelt 2006.

[vii] It is, for instance, not sufficient to rebaptize objects *a, b, c* ... in order to undertake a formalization, if a rational transformation syntax is missing. By this criterion, hence, the infertility of some classical formalization attempts in semiotics becomes understandable; e.g. Hjelmslev's ambitious algebra of glossematics (1975) which did not permit transformational possibilities of any larger interest.

[viii] I prefer to count such sign use as diagrammatic, notwithstanding some Peirce's more strict definitions demanding the presence of explicit intentional diagrams. This definition conflicts with other descriptions of diagram use, e.g. his characterization of mental imagery experiments as diagrammatic or his ideas of grammar as a type of diagrams, and is closer to his pragmatic *in actu* -requirement for sign use. I follow the latter tendency in calling icon experimentation involving rule-bound manipulation of icon parts diagrammatic.

[ix] This points to the fact that the organization of perception includes highly elaborated diagrammatic capacities without explicit conscious representation.

[x] Lakoff and Johnson's metaphor theory thus involves that structure is mapped from one domain onto another; Fauconnier and Turner's generalized ”blending” theory (comprising also non-oriented mappings) involve a schematic, so-called ”generic” space granting the coherence of the blending output.

[xi] For a thorough investigation of Kant's schematism, see Frovin Jørgensen 2005.

[xii] Diagrams as ”the main if not the only way we acquire new knowledge of relationships” have been acknowledged as a Peircean doctrine by Johansen 1993, 99.

[xiii] It must be added, though, that Peirce's attitude towards Kant's famous distinction is not unanimous. In his early and middle period, Peirce simply sees analyticity as identical to deductive necessity, while syntheticity covers ab- and inductive probability (cf. for instance Peirce's discussion with Dr. Carus in 6.595 (1893), see Otte 1997 353f), thus pertaining to ideal and real realms, respectively. Consequently, mathematics is taken to be analytic – in contrast, of course, to Kant. Later, the issue becomes more muddled, not less because Peirce now only rarely refers to the analytic/synthetic concepts explicitly. Here, theorematic reasoning – corresponding to the experiment attitude in diagram manipulation – is generally seen as synthetic. We shall return to the issue in more detail in ch. 8.

[xiv] Correction in the quote made from Robin (293), p. 59; NEM IV has "represent*ed* existential or experiential peculiarities"

[xv] The distinction between pure and applied diagrams roughly corresponds to Kant's distinction between a priori and a posteriori schemata.

[xvi] Barwise and Etchemendy highlight this important feature in diagrammatic modeling: "5. Every possibility (involving represented objects, properties, and relations) is representable. That is, there is no possible situations that are represented as impossible. 6. Every representation indicates a genuine possibility." (1995, p. 215).

[xvii] But doesn't this example run counter to Peirce's observation that the grammar of natural language is diagrammatic? No, because the contents of the words "round" and "square" are not defined by grammar. The diagrammaticality of (parts of) natural

language syntax rather lies in its instantiantion of some basic logic and ontological categories (argument structure, subject/predicate structure, etc.) It is important to remember - cf. our painting example above - that concrete signs may possess both diagrammatic and non-diagrammatic aspects, just like they may be composed of differently defined diagrams, the relation between which need not in itself be diagrammatic. Some of the strength of natural language probably lies in precisely this: it freely unites diagrams on different levels (expression, grammar, lexical semantics of the different word classes, narratology), the relative independence of which constitute language's plasticity and its ability to talk about many things, including impossible objects.

[xviii] Of course, this requires that the diagram is consistent. But the very syntax of a diagram forces it to be consistent: it is impossible to draw a square circle. This does not imply, however, that it may not be in many cases rather or extremely difficult to determine whether a given diagram is in fact consistent. For instance, an equation - a subspecies of algebraic diagrams - may hide an inconsistency very difficult to ascertain at first glance but which requires lot of work to determine: if you can derive a contradiction from it (the reductio ad absurdum method), then it is false (if we do not admit intuitionist logic etc.). The seminal difference is that you cannot derive from the grammar of the symbolic expression "a square circle" an analogous contradiction, in order to do so, you have to attempt to make a diagram of its content.

[xix] Yet, this distinction is in many cases impossible to draw beforehand, so to speak - cf. for instance the fact that a certain amount of empirical data shows up to yield a Gaussian distribution: on a first glance, this result may be conceived of as an empirical law, but it might hide a deeper law, yet uncovered, which would rather make the distribution a logical result of general mathematical principles.

[xx] This icon-index distinction in Peirce of course refers back to Kant's contention that existence (haecceity, referred to by an index) is no predicate (quality, referred to by an icon), just like it refers forward to Kripkean reference theory's rigid designators (as a certain class of indices).

[xxi] Many basic proofs in mathematics may be represented in more or less immediately accessible visual diagrams, see e.g. Nelsen 1993.

[xxii] See Misfeldt 2006 for an empirical investigation emphasizing the change between different representations of the same object in mathematical thought.

[xxiii] Our description of the workings of such experiment processes might give the idea that they are psychological and thus dependent upon a person's psychological grasping of the diagrams. This is not, however, the case. It is a crucial part of Peirce's pragmatism that it shares a basic anti-psychologism with Husserlian phenomenology (cf. ch. 6). Pragmatism insists that it is possible actions on diagrams which count – but such actions need not be performed by psychological means (cf. Peirce's notion of mind being much broader than that of psyche). Diagram operations are, by their very nature, purely formal and does not owe their validity to the psychology of those performing them. If the description of such processes may in some cases sound as if informed by psychology, this is only for the sake of understanding. When talking about the "imaginary moment" as a phase in diagram manipulation, this of course refers to the psychology of the manipulator, but the decisive thing is that this moment is made possible by structural iconicity between diagram and

object – not by the psychology of he or she who contemplates that iconicity. The case is parallel to when Peirce himself refers to human minds as bearer of signs, but immediately adds that this is only a pedagogical ”sob to Cerberus” to make his own conception easilier understandable (Letter to Lady Welby 23 December 1908, EPII, 478). The validity of diagrammatical representations in general depend just as little on psychology as does the special case of logical formalisms.

[xxiv] Peirce makes a distinction making this understandable – between corollaries and theorems. The former are propositions directly read from a diagram; the latter propositions only to be found after some more or less “ingenious” experiment. The distinction is valid, but can not be sharp: there is a continuum between, say, measuring a distance on a map; measuring the same distance with corrections according to the map projection used; constructing that projection; proving that the geometry of the surface of a sphere is isomorphous to a non-Euclidean geometry ... We return to this distinction below.

[xxv] It must be added that the truth of Kekulé’s discovery story is a matter of ongoing debate in the history of science and has not yet been definitively settled. The story is only recorded by Kekulé himself in 1890, in a celebration speech 25 years after the discovery was published. The case is even more complicated from the fact that the German Chemists’ Society at a conference in 1886 published a mock-periodical in which they appear not as the deutsche but as the ”durstige chemische Gesellschaft”. In this joke, the Benzene ring is depicted with the Carbon atoms as six apes grapping each others’ arms and legs (playing on the similarity between ”Affe” and ”Affinität” (ape and affinity) in German). Hence, it is argued, Kekulé’s 1890 memory might have been influenced by this recent joke. Thus, the story was an object of doubt rather early, and already in 1927, Kekulé’s son stated in an article that according to his childhood recollections, his father had often told the story many years before it appeared in print without ever referring to any apes – thus adding to its probability (Sposel and Rathsmann-Sponsel 2000). Also the source of the ”Uroboros”- motive – the snake biting its own tail - is discussed, and maybe traced to a sign on a pharmacy door which Kekulé remembered (ibid.). On the other hand, it has been pointed out that the German chemist Josef Loschmidt was a forerunner of Kekulé because he had, already in 1861, described a long series of organic molecules as involving ring-shaped Carbon structures (albeit not the simplest one, Benzene). Kekulé knew Loschmidt’s work as is evident from his dismissive references to it already in the same year where he refuses the shapes given by Loschmidt to have any connection to real molecule shapes (Bader and Parker 2001). This thus forms a strong argument that Kekulé’s 1863-65 discovery may have been influenced by his reading of Loschmidt some years earlier – but, on the other hand, it does not prove this influence may not have appeared in the guise of the half-dreamt snake ring of the original anecdote.

[xxvi] Ernst Cassirer’s concept of “symbolic pregnance” may be interpreted as referring to such cases of ‘spontaneous proto-diagrams’.

[xxvii]Another example would be maps with high direction sensivity but no metric, e.g. maps of the starry sky as seen from the earth; distances on this map measured in minutes and seconds of arc do not refer to real distances between stars in the universe, while directions do refer to real orientations in space.

[xxviii] Hintikka's distinction is most thoroughly presented in the papers collected in the volume *Lingua Universalis vs. Calculus Ratiocinator* (1997).
[xxix] In the logic-as-universal-language tradition, Peirce's distinction will be invisible, because any chain of reasoning will here be represented as a valid, finished symbol string *post hoc*, so that the theorematic, constructive part may merely be added as further premisses among others to the inference at issue. Peirce's distinction, however, becomes crucial because the viewpoint of the logic-as-calculus stance rather envisages the issues *ante hoc* – logic deals with the solution of problems and thus displays a continuity with heuristics and theory of science which is absent in the language tradition. When seen from the problem rather than from the solution aspect, Peirce's distinction suddenly becomes pertinent: given a set of premisses, it is of huge importance how and what to construct on their basis in order to reach a desired result.
[xxx] It is not so easy, however, that the universal-language and the calculus tradition are simply anti-iconic and iconic, respectively. Hintikka, e.g. places Hilbert and Goodman firmly in the calculus tradition. Hilbert is, according to Hintikka, no formalist, rather a fore-runner of model theory and, as discussed later, much less anti-iconist than often assumed, while Goodman the staunch anti-iconist is a calculus supporter because of his plurality of languages. On the other hand, the position of being universal-language and iconist at the same time is also possible – cf. the younger Wittgenstein with his picture theory of language maintained at the same time as the ineffability of semantics, famously making it impossible actually to point out any particular examples of the logical atoms claimed to found the theory of *Tractatus*. So even if Hintikka's calculus-language distinction is indeed orthogonal to the iconist-anti-iconist distinction, the combination of universal-language and anti-iconism is strong (and stronger than the combination of universal-language and iconism) in both analytic and continental traditions.
[xxxi] See some preliminary remarks in May and Stjernfelt 1996.
[xxxii] The German mathematician Moritz Pasch explicitly noticed this geometrical error and proposed a pure geometry in terms of purely formal manipulation of symbols with no regard to their intuitive signification, an idea that was fully developed by his famous pupil David Hilbert's formalism.
[xxxiii] The concept of "symbol" has a history so confused that it almost ought to be completely discarded; in any case, any use of it should be explicit about the precise signification intended, cf. Sørensen 1963. In formalism, symbols are arbitrary, simple signs to be manipulated syntactically; in Peirce they are not necessarily simple, and dependent on iconic meaning and indexical reference. On the symbol concept in the Kantian tradition, see my "Die Vermittlung zwischen Anschauung und Denken" (2000).
[xxxiv] Greaves, in turn, quotes from a paper by Michael Hallett who provided translation and italics.
[xxxv] Greaves has even found an amazing quote by Hilbert's close collaborator Paul Bernays in an unpublished lecture from 1921, where the stroke counting ability is directly expressed in terms of basic iconicity suggesting, as Greaves says, "a distinctly Peircean explanation":
"The philosopher is inclined to speak of this representation [between sign and number] as a relation of meaning. However, one should note that, in contrast to the usual relation between word and meaning, there is [in this example] the essential difference that *the*

object doing the representing contains the essential properties of the object to be represented. Thus the relations which are to be investigated between the objects represented *are to be found in* the objects doing the representing, and thus can be established through consideration of these." (190-191; translated by Michael Hallett, emphases by Greaves). The intuition necessary for the metamathematical finitism is hence (strongly restricted, it must be admitted) iconicity. Still remains, of course, the Hilbertian distinction between these finite calculi and the potentially infinite objects they may be taken to refer to.

xxxvi But even if we grant the basic iconicity of any "symbolic" calculus, a Peircean approach will still be faced with the problem of evidence in cases where the "imaginary moment" is precluded or where it simply refuses to appear, cf. for instance the discussion of the computer proof of the 4-color map theorem of topology which - because of its enormous size - is hard to understand as an ordinary proof which a skilled reader may adorn with interpretations from beginning to end. In proofs of this type, the trust is put in the infallibility of the computer: each step in the proof is logically valid, ergo the whole proof is valid, even if nobody has ever *observed* its truth in Peircean evidence or in Husserlian "kategoriale Anschauung".

xxxvii 3.467, from "Grand Logic," 1893.

xxxviii Life as such seems formally to involve simple diagrams known as "categorical perception," see ch. 9-12.

xxxix "... il n'y a science qu'à partir du moment où on peut plonger le réel dans le virtuel" (Thom 1989, p. 69)

xl Of course, this counterfactuality is easier to hide in experimental sciences where diagram experiments may, in many cases, be verified by similar experiments on the object itself. When this possibility falls away, counterfactual speculation prevails, cf. for instance cosmology or issues like the origin of life and origin of language. It is interesting to note, however, that the insight in the connection between counterfactual constructions and scientificity is taken up in non-experimental sciences in recent years. For instance historiography, so long trapped in a positivist determination to record only what actually happened, now seems (through inspiration from, among others, chaos theory and the formal concept of phase space in general qualitative dynamics) to realize that the actual event is only made intelligible through its juxtaposition with a rational idea of what would have happened if some central factors in the initial conditions of the situation were changed (Ferguson 1997).